PAFs for imperfect optics
(e.g., but not only, Solar Power Arrays)

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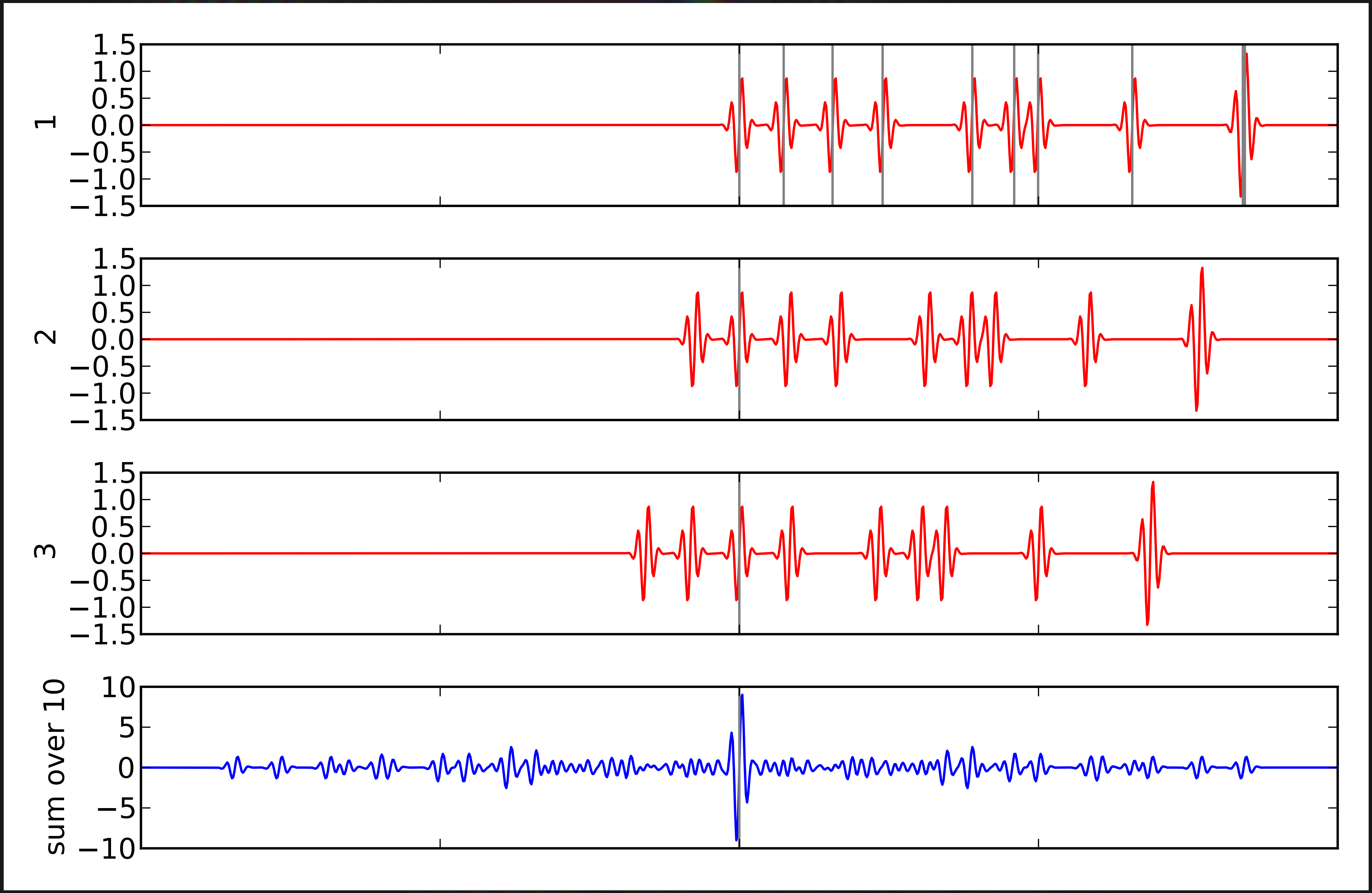
PAFs for imperfect optics

- Solar power arrays: why PAFs are needed
- SNR considerations
- practical issues
- imperfect optics
- Effelsberg PAF
- mitigation of non-linear RFI effects

The Solar power array problem

- n mirrors, different delays
- signal spread over larger area
- how to deal with delay/phase differences?
- can we catch the signal with one big feed?
- re-aligning approach (Alan Roy)
  - ⋆ feed sees n shifted copies of signal
  - ⋆ compensate for n delays
  - ⋆ sum up all n re-aligned signals

Single-feed re-aligning approach



Sensitivity for single-feed system

- for the moment: assuming good focus per mirror
- re-aligned sum in comparison to one mirror
  - ⋆ signal voltage $\times n \rightsquigarrow$ power $\times n^2$
  - ⋆ noise power $\times n$ (incoherent)
  - ⋆ SNR per sample given by $\dfrac{\text{signal power}}{\text{noise power}}$
  - ⋆ SNR scales with n
- should be as good as n mirrors in phase !?
- too good to be true
- first understand standard single-feed system

Towards understanding single-feed standard system

- paradox

  How does the power in the focus scale if we double the area of the mirror?

- received power $\propto$ collecting area

$\rightsquigarrow$ power $\times 2$

- field in focus $\propto$ field integrated over mirror

$\rightsquigarrow$ integrated field $\times 2$, power $\times 4$ (?)

- Cannot both be true, right?

Resolving the paradox

- field argument is about power density
- which area?
- focal spot (Airy disk) scales with $1/A_{\text{coll}}$
- power from field argument $\times 4 \times 1/2 \rightsquigarrow \times 2$

$\rightsquigarrow$ size of feed must be matched

- $A_{\text{feed}} \gtrsim \dfrac{\lambda^2 f^2}{A_{\text{coll}}}$ to collect all radiation
- $A_{\text{feed}} \lesssim \dfrac{\lambda^2 f^2}{A_{\text{coll}}}$ to ‘illuminate’ entire mirror

Feed size for incoherent mirrors

- $A_{\text{feed}} \gtrsim \dfrac{\lambda^2 f^2}{A_{\text{coll,single}}}$ to collect all radiation from individual mirrors (or resolve mirrors from each other)
- $A_{\text{feed}} \lesssim \dfrac{\lambda^2 f^2}{A_{\text{coll,field}}}$ to 'illuminate' entire mirror field
- matched feed not possible
- assume small feed (does not matter within this range)
  - ⋆ sees fraction $\dfrac{A_{\text{coll,single}}}{A_{\text{coll,field}}} \sim \dfrac{1}{n}$ of available power
  - ⋆ compensates re-alignment scaling with n

⇝ **not better than single mirror with matched feed!**

PAF to the rescue!

[Chippendale et al. (2016), arXiv:1606.03533]

PAF approach

- matched PAF is possible!
- $A_{\mathrm{PAF}} \gtrsim \dfrac{\lambda^2 f^2}{A_{\mathrm{coll,single}}}$ to collect all radiation from individual mirrors
- $A_{\mathrm{element}} \lesssim \dfrac{\lambda^2 f^2}{A_{\mathrm{coll,field}}}$ to 'illuminate' entire mirror field
- fill entire illuminated focal area with small feeds
- fine sampling to resolve speckles
- need $\sim A_{\mathrm{coll,field}}/A_{\mathrm{coll,single}} = n$ feeds (thousands!)

$\rightsquigarrow$ Solar power array with PAF should work!

Towards SNR of beamformed signal

- voltages are (complex) Gaussian noise
- voltage variance = mean power = power rms (per Nyquist sample)
- (spectral) signal power P, noise power P_0, assume $P_0 \gg P$
- per element and sample: $\mathrm{SNR} = \dfrac{P}{P_0}$
- uniform intensity distribution
  - ⋆ N elements, each with signal P and noise P_0
  - ⋆ summed voltage $\times N \rightsquigarrow$ power $\times N^2$
  - ⋆ independent noise adds incoherently: noise power $\times N$

$$\rightsquigarrow \mathrm{SNR} = N\frac{P}{P_0} = \frac{P_{\mathrm{tot}}}{P_0}$$

SNR of beamformed signal: non-uniform speckles

- N elements, each with signal P_j and noise P_0
- voltage weights w_j (here without phases)

$$\mathrm{SNR} = \frac{\left(\sum w_j \sqrt{P_j}\right)^2}{\sum w_j^2 P_0}$$

- optimum weights $w_j \propto \sqrt{P_j}$

$$\boxed{\mathrm{SNR} = \frac{\sum P_j}{P_0}}$$

- SNR only depends on total collected power

$\rightsquigarrow$ **Solar power array + large PAF ≈ proper telescope!**

- (neglecting noise correlations between elements)

Practical issues

- heat (use dedicated tower?)
- RFI
- mirrors: do they reflect radio waves?
  - ⋆ must be thicker than skin depth
  - ⋆ ca. 0.5 – 2 μm for 10 – 1 GHz
  - ⋆ metal mirrors (Gemasolar) okay
  - ⋆ frontside coated mirrors too thin
  - ⋆ what about standard bathroom mirrors?
  - ⇝ may actually work!
- PAF with 1000 – 100 000 elements expensive
  - ⋆ can it be made cheaper?
  - ⋆ **may still be cheaper than traditional SKA2 !**

Imperfect optics

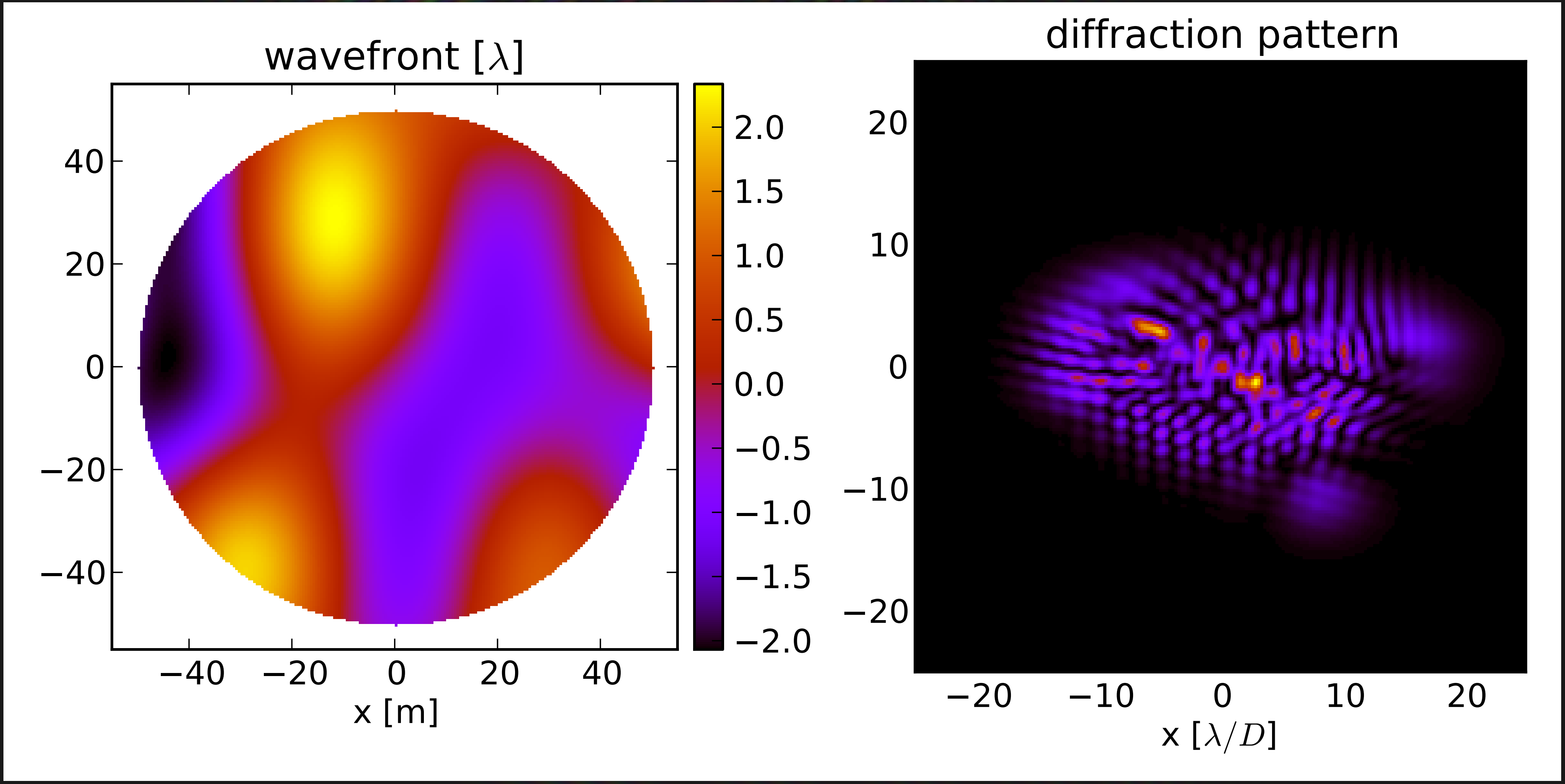
- PAF works for
  - ⋆ good optics: beam $\approx$ element
  - ⋆ Solar power array: large field of speckles
  - ⋆ as aperture array: Fourier transform
- can also correct for moderate optical aberrations
- why not use it for bad optics?
- e.g. frequencies beyond design specs

Quantifying the PAF size

- typical wavefront error Δ (not $\ll \lambda$)
- over typical distance L
- deflection in focus: $\Delta/L \times f$
- size of Airy disk ($\sim$ speckle size): $\lambda/D \times f$
- field size in units of speckles: $\frac{\Delta}{\lambda}\frac{D}{L}$
- simulations: $D = 100\,\mathrm{m}$ (Effelsberg)
- various Δ and L

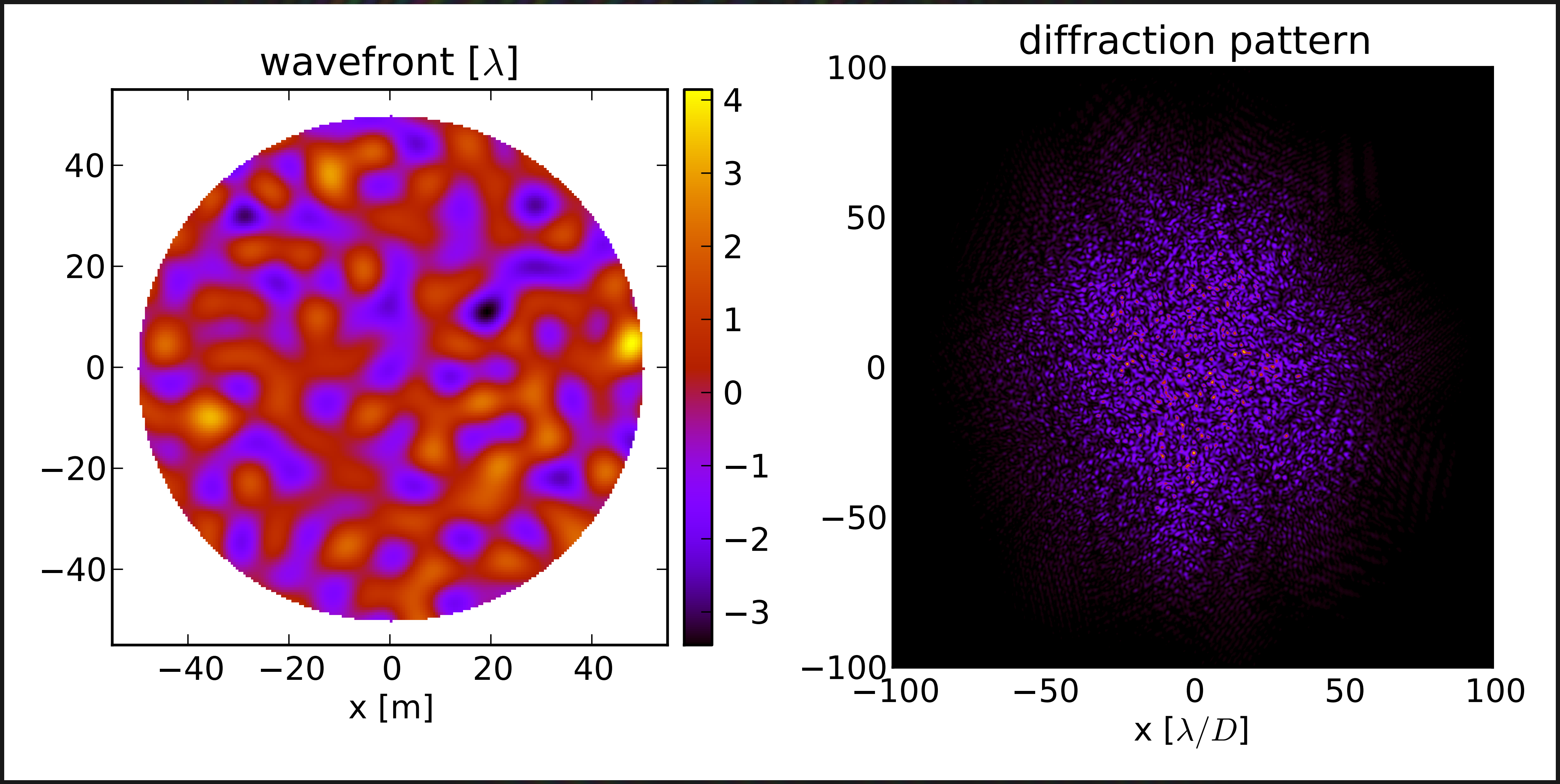
Simulations 1

- wavefront rms $= 1\,\lambda$
- $L \approx 12\,m$ (defined as rms / typical gradient)



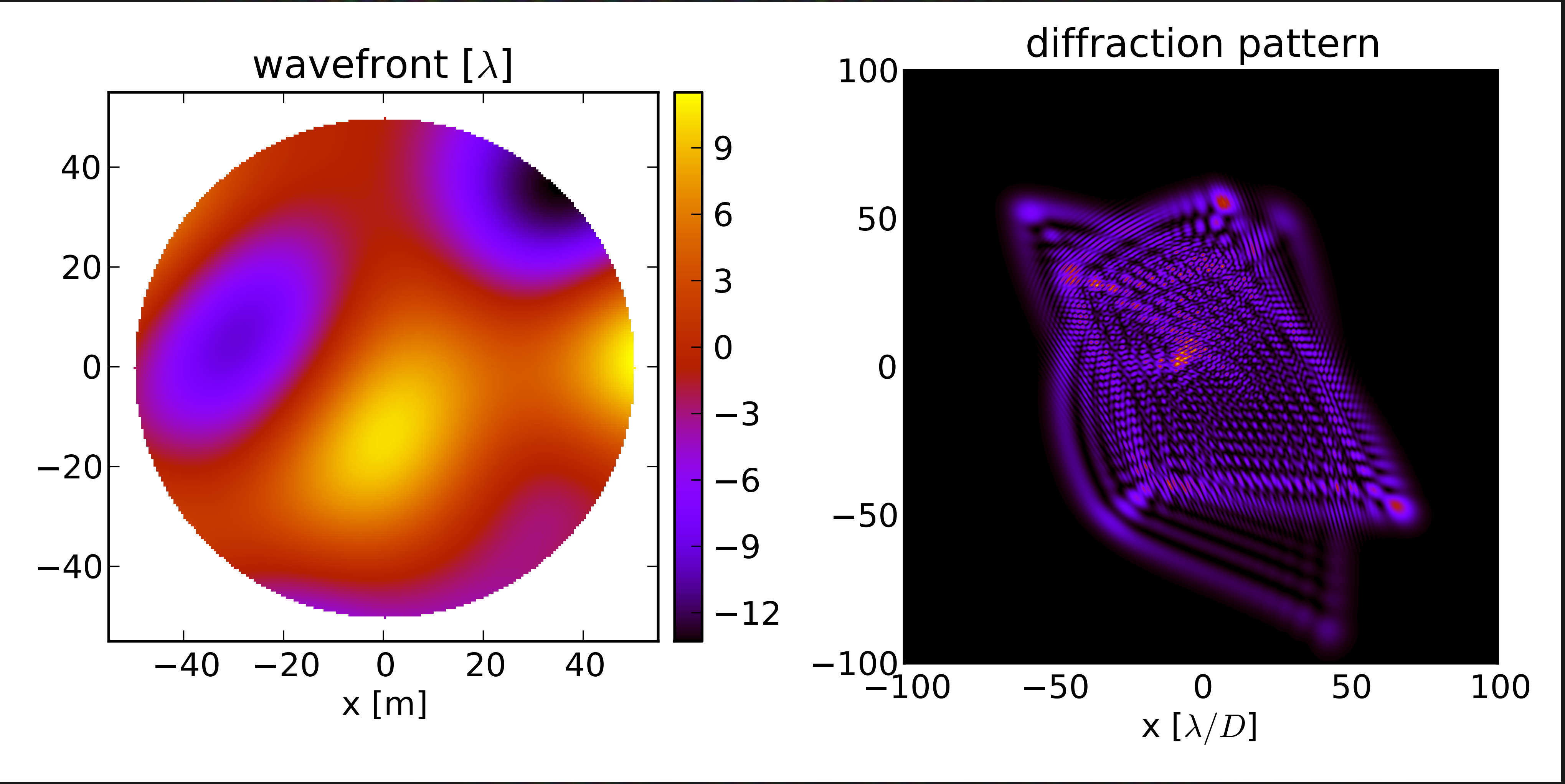
Simulations 2

- wavefront rms $= 1\,\lambda$
- $L \approx 2.4\,m$

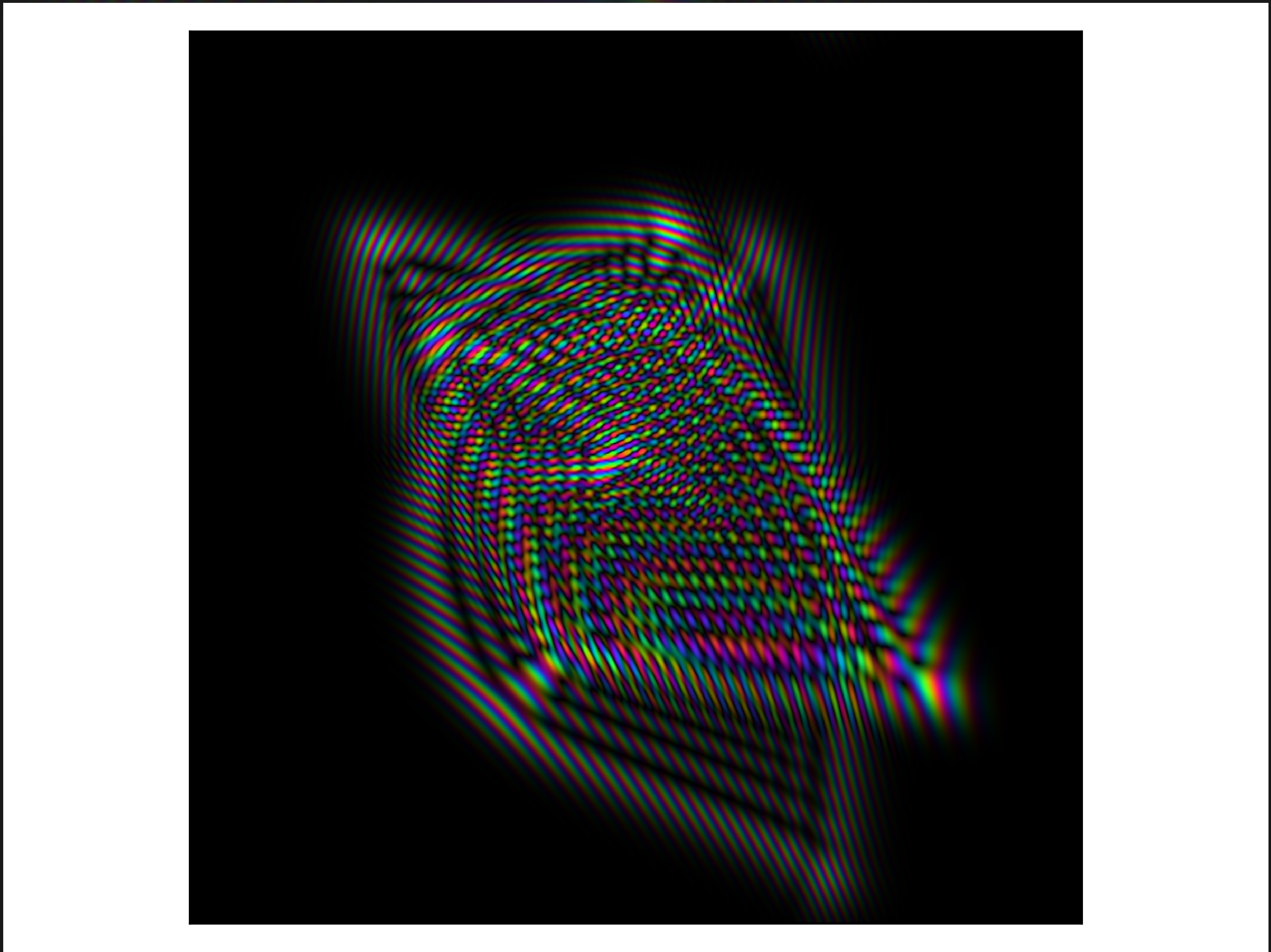


Simulations 3

- wavefront rms $= 5\,\lambda$
- $L \approx 14\,m$



Speckle pattern with phases



conclusions: PAF for Solar power arrays

- single-feed system not good for SNR
- PAF should work with full SNR, but expensive
  (not as expensive as traditional SKA?)
- also good for big cheap, dented dishes! mm with Effelsberg?
  (quickly gets expensive)
- interferometry possible
  - ⋆ between solar power arrays
  - ⋆ within solar power arrays (advanced beamforming)
- Very promising! Experiments in Jülich?

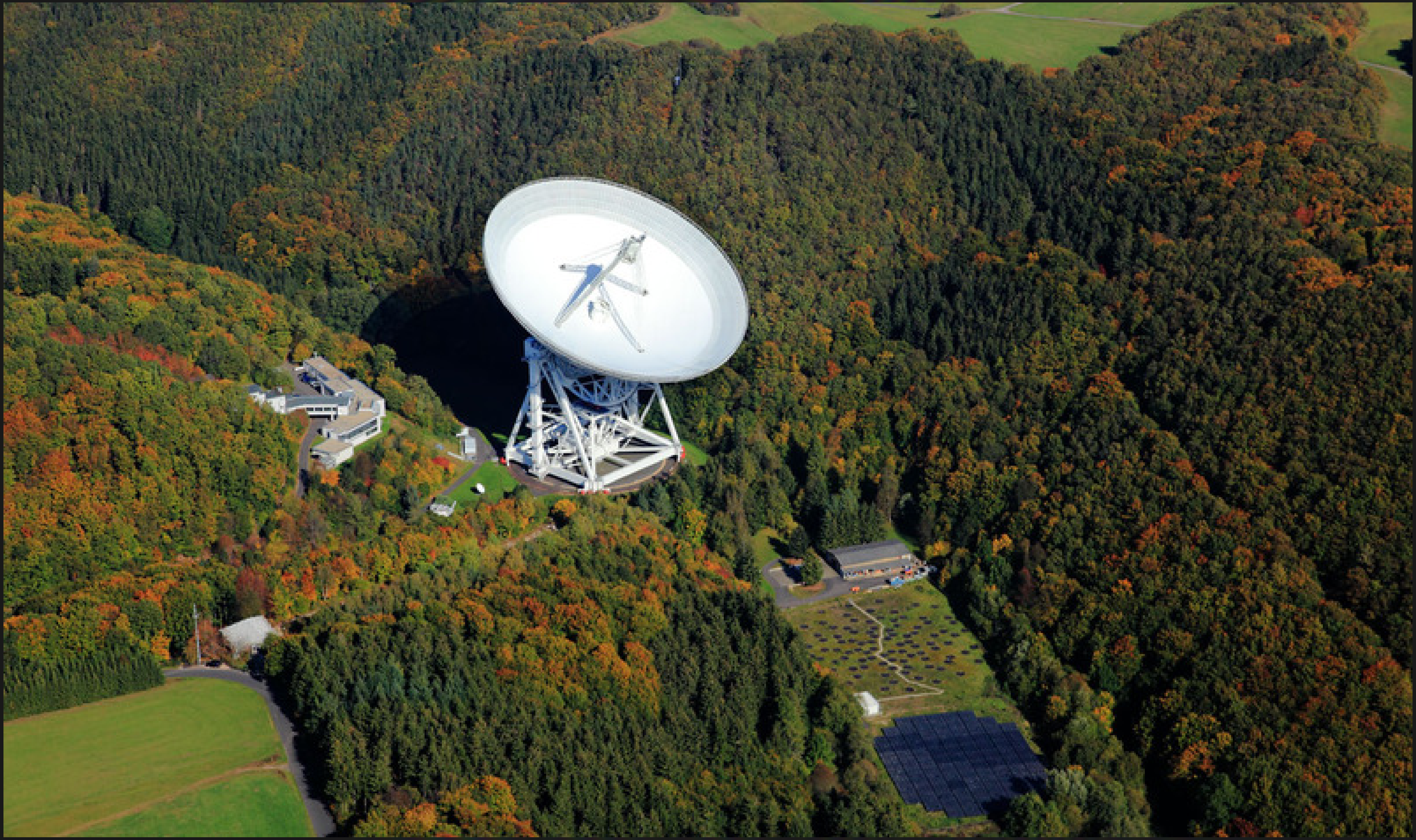
Solar Tower Jülich

Solar Tower Jülich: research platform

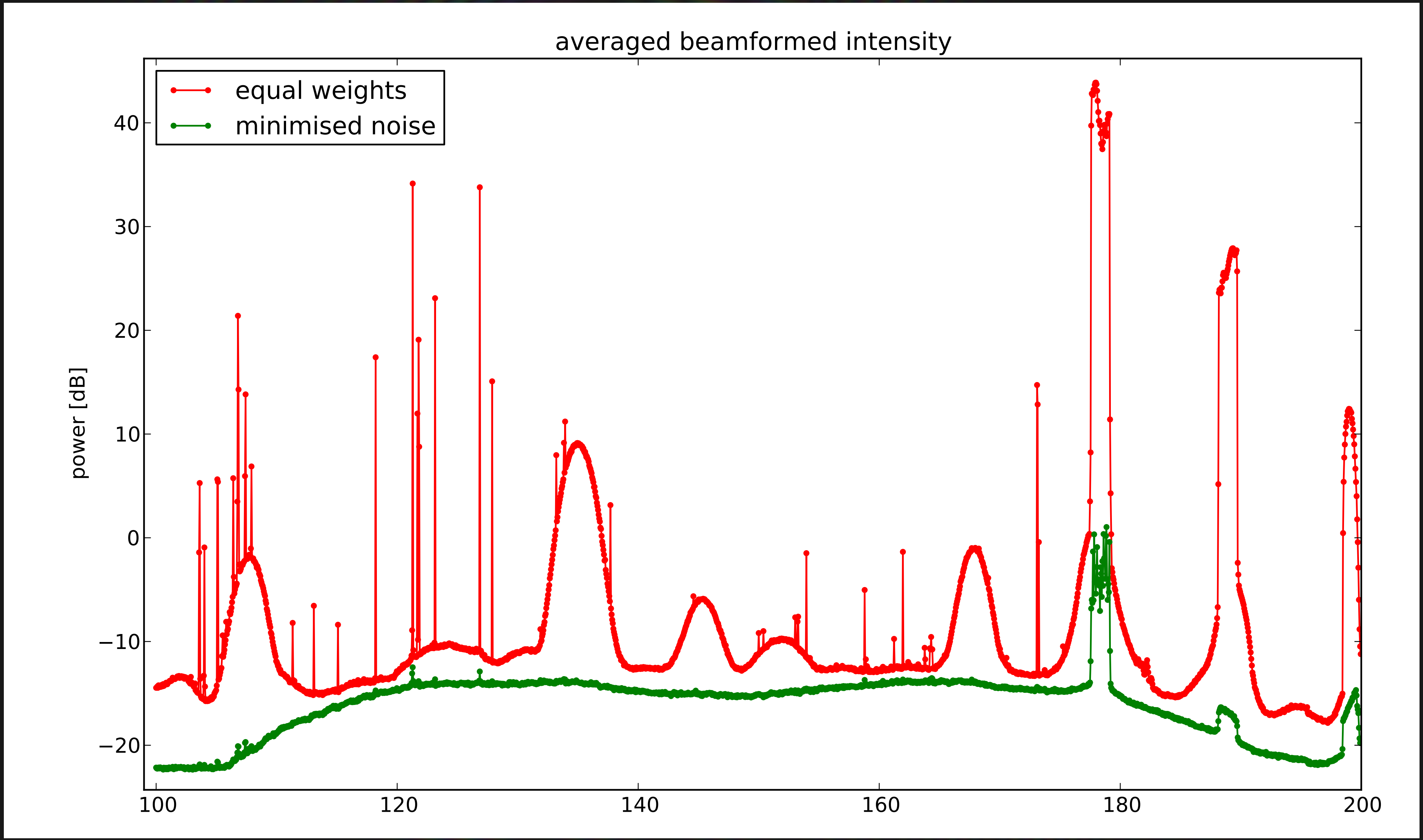
Effelsberg PAF

- Australian MkII checkerboard PAF for 100 m Effelsberg
- currently commissioned at Parkes (talk A. Chippendale)
- backend currently bottle neck
- online data processing for FRBs and pulsars
- move to Effelsberg later 2016
- additional difficulties there
  - ⋆ strong RFI, possibly saturating
  - ⋆ non-linear effects no show-stopper!
  - ⋆ 1 MHz bands maybe too wide for RFI excision

Effelsberg 100 m and LOFAR

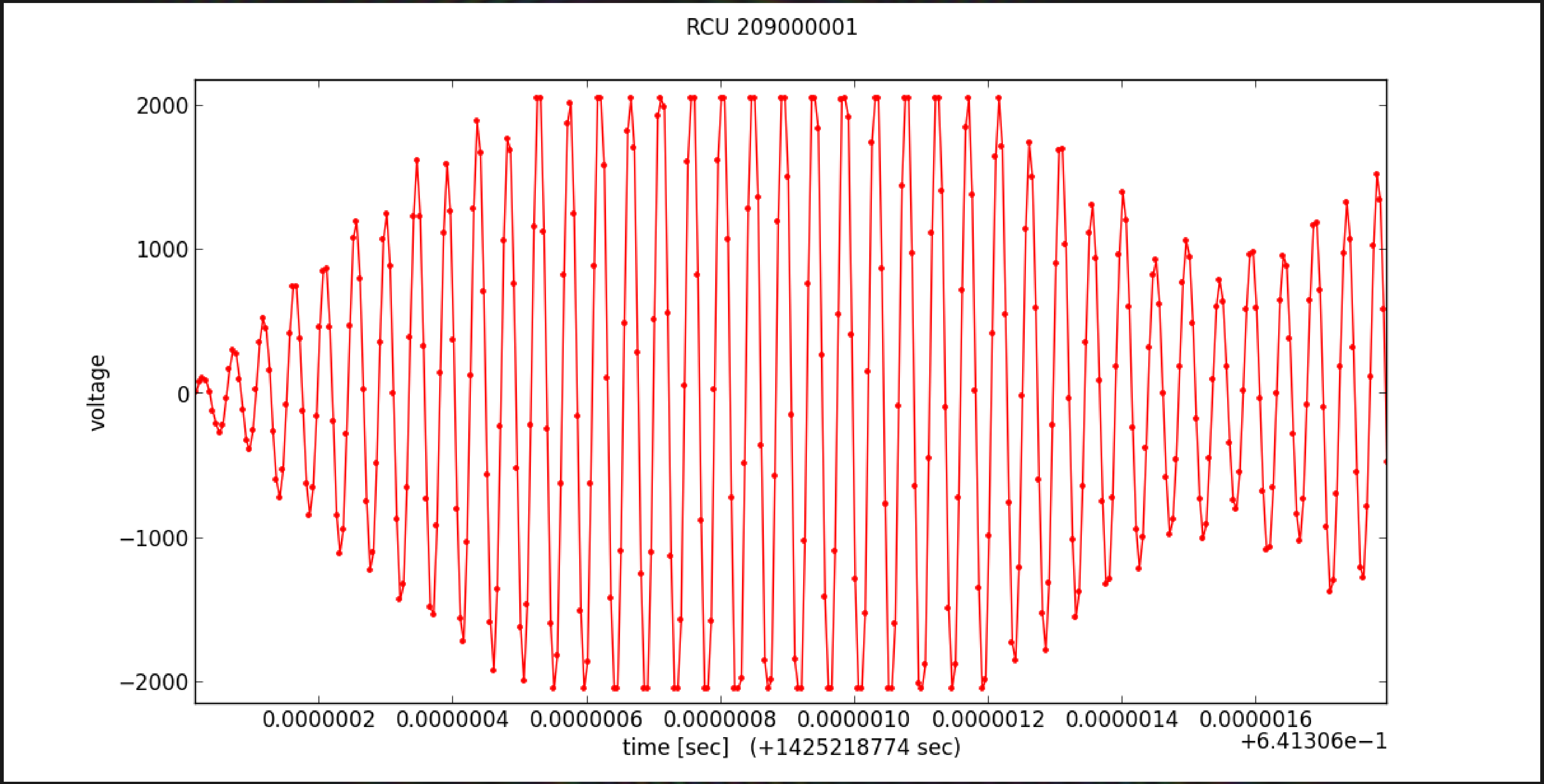


RFI mitigation for LOFAR (Norderstedt station)



freq [MHz]

ADC clipping (LOFAR Norderstedt station)



Mitigation of non-linear RFI products

- beamforming formalism is linear
- why does maximum-SNR beam reduce intermods?
- monochromatic signal $f(t) = f_0 \, e^{2\pi i \nu t}$
- non-linear response $r(t) = \sum_k a_k f_0^k e^{2\pi i k \nu t}$

⇝ additional 'independent' signals at other frequencies

- similar for intermodulation products

⇝ is not linear, but can be treated as linear (to be published)

- This is good news for Effelsberg PAF!

Summary

- Solar power array radio telescope may actually work
- PAF is essential
- also good for bad optics or too high frequencies
- PAF will come to Effelsberg soon
- RFI situation difficult, but there is hope

$\rightsquigarrow$ **PAFs good for many non-standard applications, much more interesting than just many beams!**