Fast Numerical Methods for the Simulation Focal Plane Array

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Next-generation Radio Telescopes: SKA

1937

Grote Reber, built a parabolic, 9.5-m diameter, reflector dish in his backyard

2016

SKA antennas will extend over thousands of Kilometers in SA and Australia.
Advanced Focal Array Demonstrator

Bruce Veidt (NRC-DRAO, Canada)

Table: 3D Array specifications

<table>
<thead>
<tr>
<th>Specification</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Frequency range</td>
<td>0.7 – 1.5 GHz</td>
</tr>
<tr>
<td>Element spacing</td>
<td>λ/2 = 10 cm</td>
</tr>
<tr>
<td>Array size</td>
<td>≤ 1 m × 1 m</td>
</tr>
<tr>
<td>Element dissipative loss</td>
<td>&lt; 0.1 dB</td>
</tr>
<tr>
<td>$T_{LNA}$</td>
<td>&lt; 15 k</td>
</tr>
<tr>
<td>$G_{LNA}$</td>
<td>25 – 35 dB</td>
</tr>
<tr>
<td>Array mass</td>
<td>&lt; 50 kg</td>
</tr>
</tbody>
</table>

Goal: Simulation and design of Focal Plane Array of 71 antennas.
Large Focal Plane Array

- 5 × 7 antennas polarized along x axis
- 6 × 6 antennas polarized along y axis.

Array periodic structure construction

Connecting basis functions
Goal: Large array analysis with Method-of-moments

Idea: Compress the MoM matrix using set of current distributions from the solution of smaller problems.
ASM-MBF Principle

Simulation of smaller problem
Array Scanning Method (ASM)
Small Finite Array.

Current Extraction
Extract L current distributions
\[ \vec{J}_i = \vec{Q}_i \vec{f} \]

Compress & Solve System
- Express current distribution as linear combination of the obtain set of currents from smaller problems.
- Compress the system matrix using \( Q_i \).
- Solve the compressed system.
- Decompress the current solution.

Infinite simulation

Finite simulation
ASM-MBF Formulation

**Set of current distributions**

\[
\tilde{Q}_i = \begin{bmatrix} \tilde{Q}_\text{ASM} & \tilde{Q}_\text{small array} \end{bmatrix}
\]

All the sets of current distributions are concatenated in \( Q_i \) matrix.

**Linear Combination**

\[
\tilde{x}_i = \tilde{Q}_i \tilde{y}_i
\]

Then any current distribution can be expressed as a linear combination of these current sets.

**Compress & Solve System**

The MoM system of equation is compressed as shown:

\[
\begin{pmatrix}
Q_1^T & \cdots & 0 & 0 \\
0 & Q_2^T & \cdots & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & \cdots & Q_M^T
\end{pmatrix}
\begin{pmatrix}
\bar{Z}
\end{pmatrix}
= \begin{pmatrix}
Q_1 & \cdots & 0 & 0 \\
0 & Q_2 & \cdots & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & \cdots & Q_M
\end{pmatrix}
\begin{pmatrix}
y_1 \\
y_2 \\
\vdots \\
y_L
\end{pmatrix}
= \begin{pmatrix}
Q_1^T & \cdots & 0 & 0 \\
0 & Q_2^T & \cdots & 0 \\
0 & 0 & \ddots & 0 \\
0 & 0 & \cdots & Q_M^T
\end{pmatrix}
\begin{pmatrix}
\bar{\nu}
\end{pmatrix}
\]

**Size of Z** is \((N*M)\times(N*M)\)

**Size of \(Z\) compressed** is \((N*L)\times(N*L)\)

**Compression ratio** = \((M/L)\)

- \( M = 838 \)
- \( L = 28 \)
- \( \text{Cr} = 30 \)

M is the number of basis functions

L is the number of current distributions

N is the number of the array antennas
ASM-MBF Validation & Discussions

3D Bowtie antenna:
- Meshed by $M = 180$ basis functions
- And compressed by ASM $3 \times 3$, $L = 9$
- Compression ratio is 20.

We compare the CPU time:

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brute-force</td>
<td>C-code program Pentium (R) 4 CPU 3.06 GHz 1.5 GB of RAM CPU time: 4h 21min</td>
</tr>
<tr>
<td>ASM-MBF</td>
<td>Matlab program Pentium (R) 4 CPU 3.06 GHz 1.5 GB of RAM CPU time: 1h 19min</td>
</tr>
</tbody>
</table>

Conclusion:
- Matrix interaction calculation represents an important share of time. This is proven to be reduced using Multipoles [2].
- This method is interesting for Very Large Array where memory resources needed to bypass the available memory capacity.
- The most important thesis of this method is the complexity reduction of the problem and the quality of the results.
- Error below $-40$ db using only ASM $(4 \times 4)$.
- This ASM-MBF method stays applicable with dielectric structures. Further works has been done in [1].

Potential of 3D TSA

Advantages of Metal only Vivaldi:

- Direct feed almost no soldering required.
- No dielectric material: dielectric loss elimination.
- Host LNA as near as possible to feed: reduced noise level.
- Highly modular -> easy upgrade of the system since each element can be treated alone.
- Easy to manufacture and mount.
- Stability and reproducibility of the array.
- Cost becomes fair for mass production.
Feed types investigations

- **Open stub feed**
- **Pin feed with series capacity**

Infinite array simulation with connecting basis functions

**Reflection coefficient**

With reference to 85 Ohms
Optimization of the 3D Vivaldi Antenna

### Reflection coefficient
- **Sweep thickness**
  - Flat
  - $h = 5\text{mm}$
  - $h = 7.5\text{mm}$
  - $h = 10\text{mm}$
  - $h = 12.5\text{mm}$

- **Sweep balun**
  - $d_{sl} = 10\text{mm}$
  - $d_{sl} = 15\text{mm}$
  - $d_{sl} = 20\text{mm}$
  - $d_{sl} = 25\text{mm}$

### Real part of Z
- **Sweep thickness**

### Imag part of Z
- **Sweep thickness**

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We extracted 16 current distributions from the ASM simulations.
Small Finite Array

- Finite Simulation

- We extracted 12 current distributions from the simulation of this array.
### 71 Antenna Array Simulation Results

**Array map**

- **E-plane**
  - Array map

- **H-plane**
  - 0.6GHz
  - 1GHz
  - 1.6GHz

**Graphs**

- **S_{1313}**
- **S_{1311}**
- **S_{1352}**
- **S_{1350}**

**Table:** Half Power Beam Width of the center element of the array.

<table>
<thead>
<tr>
<th>Frequency [GHz]</th>
<th>H-plane ((\phi = 0^\circ))</th>
<th>E-plane ((\phi = 90^\circ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>At 0.6 GHz</td>
<td>135°</td>
<td>69°</td>
</tr>
<tr>
<td>At 1 GHz</td>
<td>108°</td>
<td>126°</td>
</tr>
<tr>
<td>At 1.6 GHz</td>
<td>111°</td>
<td>111°</td>
</tr>
</tbody>
</table>
Truncation Effects at Array Borders

Array map

1GHz

(a) Hpol Antenna 11 (b) Hpol Antenna 13 (c) Vpol Antenna 11 (d) Vpol Antenna 13

(e) Hpol Antenna 50 (f) Hpol Antenna 52 (g) Vpol Antenna 50 (h) Vpol Antenna 52

0.6 GHz

(a) Hpol Antenna 11 (b) Hpol Antenna 13 (c) Vpol Antenna 11 (d) Vpol Antenna 13

(e) Hpol Antenna 50 (f) Hpol Antenna 52 (g) Vpol Antenna 50 (h) Vpol Antenna 52

1.6GHz

(a) Hpol Antenna 11 (b) Hpol Antenna 13 (c) Vpol Antenna 11 (d) Vpol Antenna 13

(e) Hpol Antenna 50 (f) Hpol Antenna 52 (g) Vpol Antenna 50 (h) Vpol Antenna 52
Simulation vs. Measurements

- Excellent agreement between the simulation and measurements.

### Vivaldi 3D dimensions
- Width $a = 10$ cm
- Height $b = 14$ cm
- Cavity diameter $c = 2$ cm
- Slot Width $d = 0.3$ cm
- Thickness = 0.5 cm

### S-parameters Simulations vs. Measurements

(a) $S_{11}$ simulation vs. measurements.

(b) $S_{12}$ simulation vs. measurements.

(c) $S_{13}$ simulation vs. measurements.

(d) $S_{14}$ simulation vs. measurements.
Rectangular Arrays Vs Circular Arrays

- Less truncation effect at the border of the array.
- Advantage of the rotation similarity of the radiation pattern.
- Polarimetric advantage using different polarizations.
- Rotational symmetry: pattern calibration is made easier.
Array Scanning Method (ASM)

**Current Distribution**

\[ I(m) = \frac{1}{2\pi} \int_0^{2\pi} I^\infty(\psi) e^{-im\psi} d\psi \]  

**Array Scanning Method**

\[ I(m) \approx \frac{1}{N} \sum_{p=0}^{N-1} I^\infty(\psi_p) e^{-im\psi_p} \]  

where \( \psi_p = \frac{2\pi p}{N} \)  

with \( 0 < p < N - 1 \)  

Finite array: direct and reflected waves  

Reflected by array ends  

Generated by single source  

Reflected by array ends

Reflected wave  

Progressive wave  

Reflected wave

Aliased through discrete ASM

Introduction  
ASM-MBF for Large Array Simulation  
Simulation of Large Rectangular Array  
ASM for Circular Array Simulation  
Simulation of Large Hexagonal Array  
Conclusion & Future Works

Goal  
Principle  
Formulation  
Application
MoM System

\[
\begin{pmatrix}
Z_{1,1} & Z_{1,2} & \cdots & Z_{1,N} \\
Z_{2,1} & Z_{2,2} & \cdots & Z_{2,N} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{N,1} & Z_{N,2} & \cdots & Z_{N,N}
\end{pmatrix}
\begin{pmatrix}
l_1 \\
l_2 \\
\vdots \\
l_N
\end{pmatrix}
= 
\begin{pmatrix}
V_1 \\
V_2 \\
\vdots \\
V_N
\end{pmatrix}
\]  
\(1\)

ASM for Circular Array

\[
\left( \begin{array}{cccc}
Z_{1,1} & Z_{1,2} & \cdots & Z_{1,N} \\
Z_{2,1}e^{j\psi_p} & Z_{2,2}e^{j\psi_p} & \cdots & Z_{2,N}e^{j\psi_p} \\
\vdots & \vdots & \ddots & \vdots \\
Z_{N,1}e^{j(N-1)\psi_p} & Z_{N,2}e^{j(N-1)\psi_p} & \cdots & Z_{N,N}e^{j(N-1)\psi_p}
\end{array} \right)
\begin{pmatrix}
l_1 \\
l_2 \\
\vdots \\
l_N
\end{pmatrix}
= 
\begin{pmatrix}
V_1(\psi_p) \\
V_2(\psi_p) \\
\vdots \\
V_N(\psi_p)
\end{pmatrix}
\]  
\(2\)

\[
\left( \sum_{m=0}^{N-1} Z_{1,m+1}e^{jm\psi_p} \right) \begin{pmatrix}
l_1 \\
l_2 \\
\vdots \\
l_N
\end{pmatrix}
= 
\begin{pmatrix}
V_1(\psi_p) \\
V_2(\psi_p) \\
\vdots \\
V_N(\psi_p)
\end{pmatrix}
\]  
\(3\)

ASM yields exact solution

\[
l(m) \approx \frac{1}{N} \sum_{p=0}^{N-1} l^\infty(\psi_p)e^{-jm\psi_p}
\]  
\(5\)

- MoM system \((N*1)x(N*M)\) solution is reduced to \(N*(MxM)\) systems
- \(N\) antennas and \(M\) basis functions to discretize each antenna,
Dense Hexagonal Array

Periodic element of the array

Dense Hexagonal Array
### Introduction

ASM-MBF for Large Array Simulation
Simulation of Large Rectangular Array
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Conclusion & Future Works

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**Outer**

#### E-plane (xOz)

1GHz

2GHz

3GHz

#### H-plane (yOz)

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**Goal**

- Principle
- Formulation
- Application

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Goal
Principle
Formulation
Application

H-plane (yOz)
E-plane (xOz)

"to be fixed"
Large Hexagonal Array
ASM-MBF for Large Hexagonal Array

2x2 ASM combined with 12 elements finite array

Current error is below 40dB
Hexagonal Vs. Rectangular Array
Large Hexagonal Array

- Single antenna
- 12 element array
- 90 elements Hexagonal Array

Infinite simulation
Finite simulation

 ASM-MBF
Center element antenna 15 Pol1

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**E-field (dB)**

**θ (Degree)**

-7dB Beamwidth = 160

**Co-pol**

**Cross-pol**

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**Goal**

**Validation**

**Hexagonal Array Results**

**Rectangular Array Results**

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**Introduction**

**ASM-MBF for Large Array Simulation**

**Simulation of Large Rectangular Array**

**ASM for Circular Array Simulation**

**Simulation of Large Hexagonal Array**

**Conclusion & Future Works**

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Rémi M. Sarkis

Focal Plane Antenna Arrays for Astronomic Applications
Center element antenna 42 Pol2

E-field(dB) vs θ (Degree)

-7dB Beamwidth=153
Center element antenna 72 Pol3

Graph showing E-field (dB) vs. θ (Degree) for Co-pol and Cross-pol with a -7dB Beamwidth of 150 degrees.
Large Rectangular Array

Single antenna

12 elements array

63 elements rectangular array

Infinite antenna

Finite antenna

ASM-MBF
Center element 16 Pol1

-7dB Beamwidth = 126deg
Center element 44  Pol2

-7dB Beamwidth=147deg
Conclusion

- We presented ASM-MBF for the simulation of Large Focal Plane Array
- Link between ASM and Block circulant matrix solution.
- Novel design of 3D Vivaldi antenna
  - Light weight of the antenna.
  - Precise fabrication technology.
  - Suitable to host LNA.
- Study of different circular array structures
  - Dense and Concentric Hexagonal arrays.
  - Easier Calibration: Radiation pattern can be compensated.