Mutual coupling analysis of line feeds devoted to cylindrical reflectors

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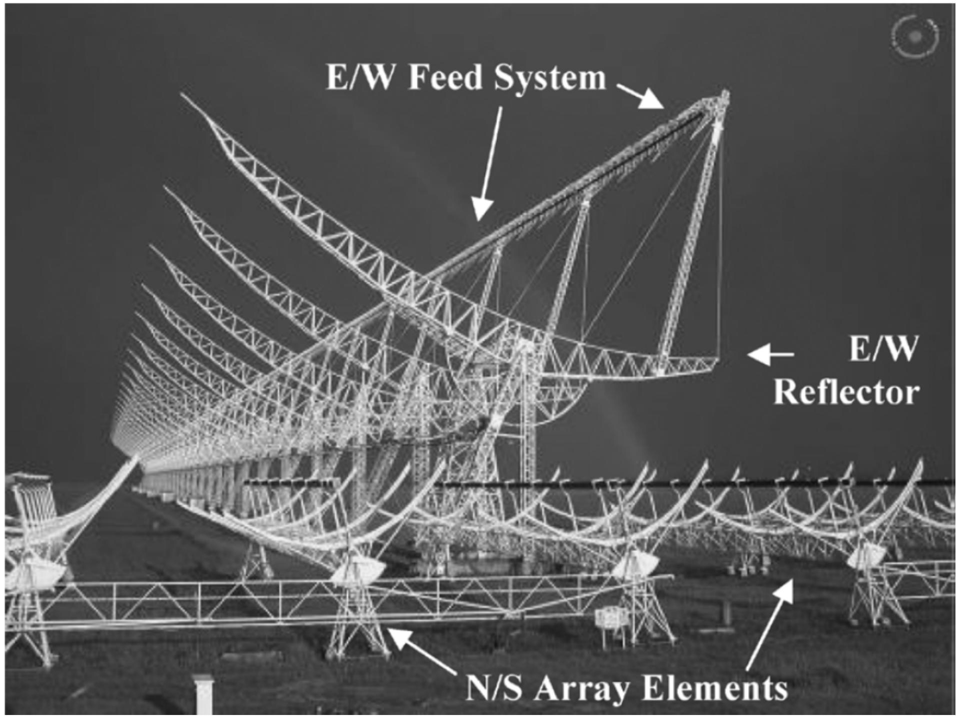
PAF WORKSHOP 2016
PHASED ARRAY FEED WORKSHOP - AUGUST 24-26, 2016 - CAGLIARI, ITALY

Line feeds for cylindrical arrays

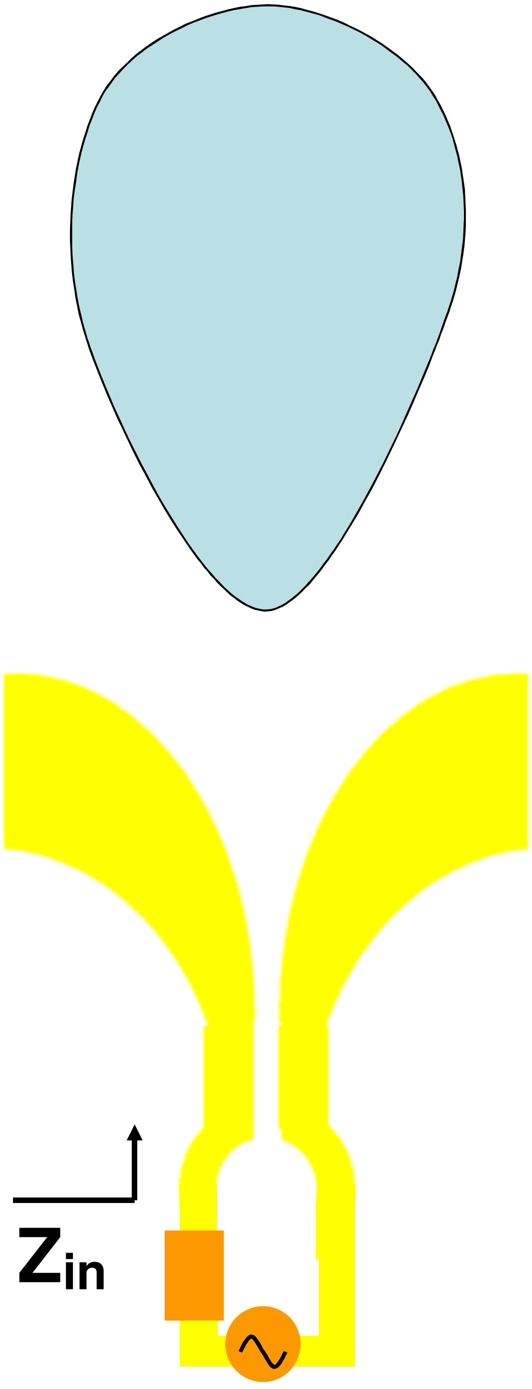


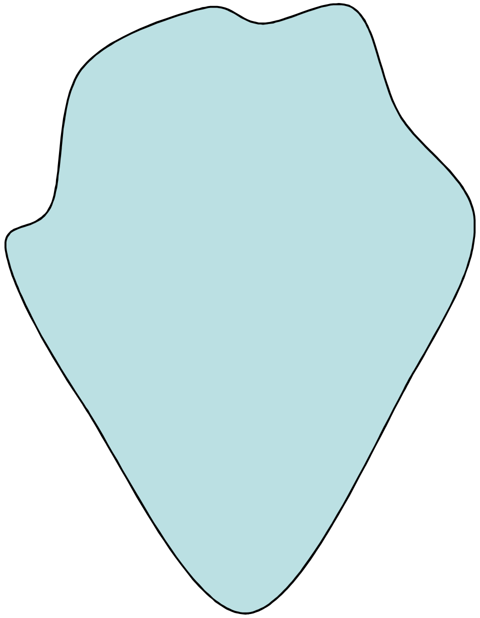
Nothern-Cross telescope, Virone et al., T-AP, June 2011

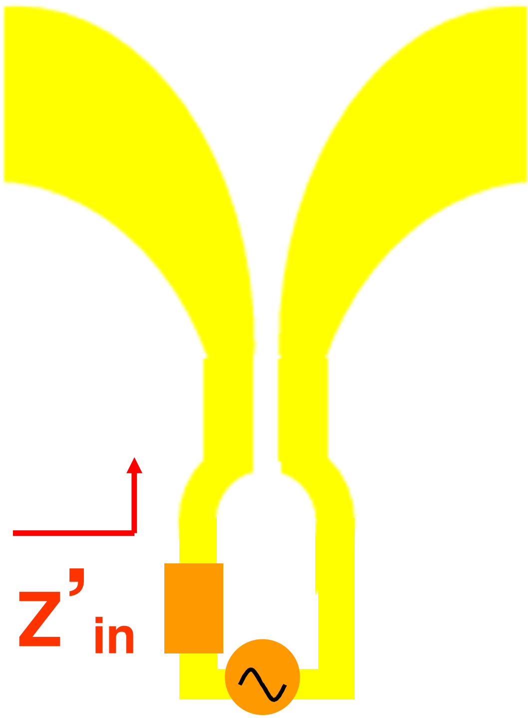
Include coupling:
- between elements in feed, with possibly several elements per unit cell,
- between feed line and reflector, involving scattering by edges,
Include noise coupling

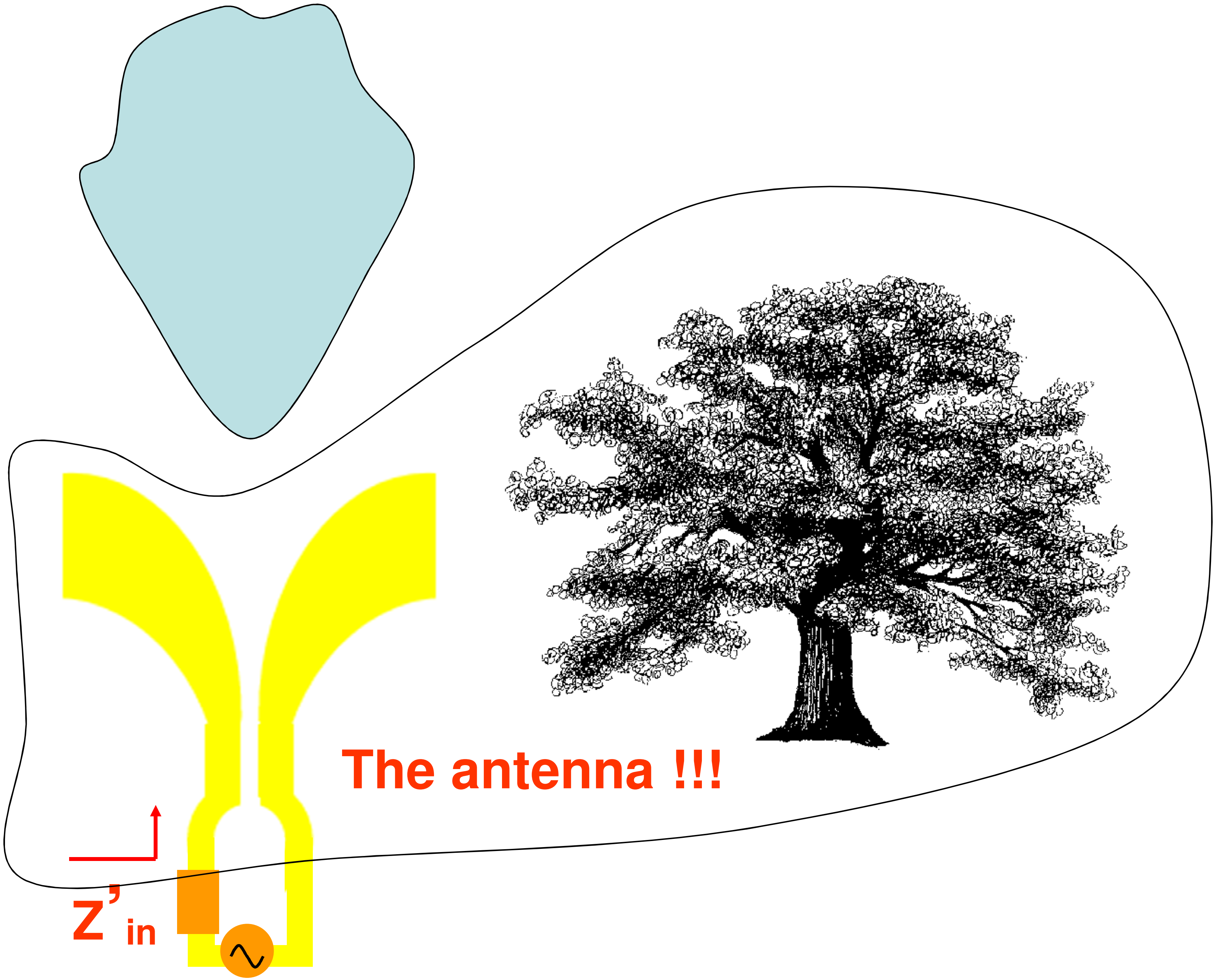
Outline

- Generalities about mutual coupling
- Analysis of cylindrical arrays
- Examples
- Challenges and prospects

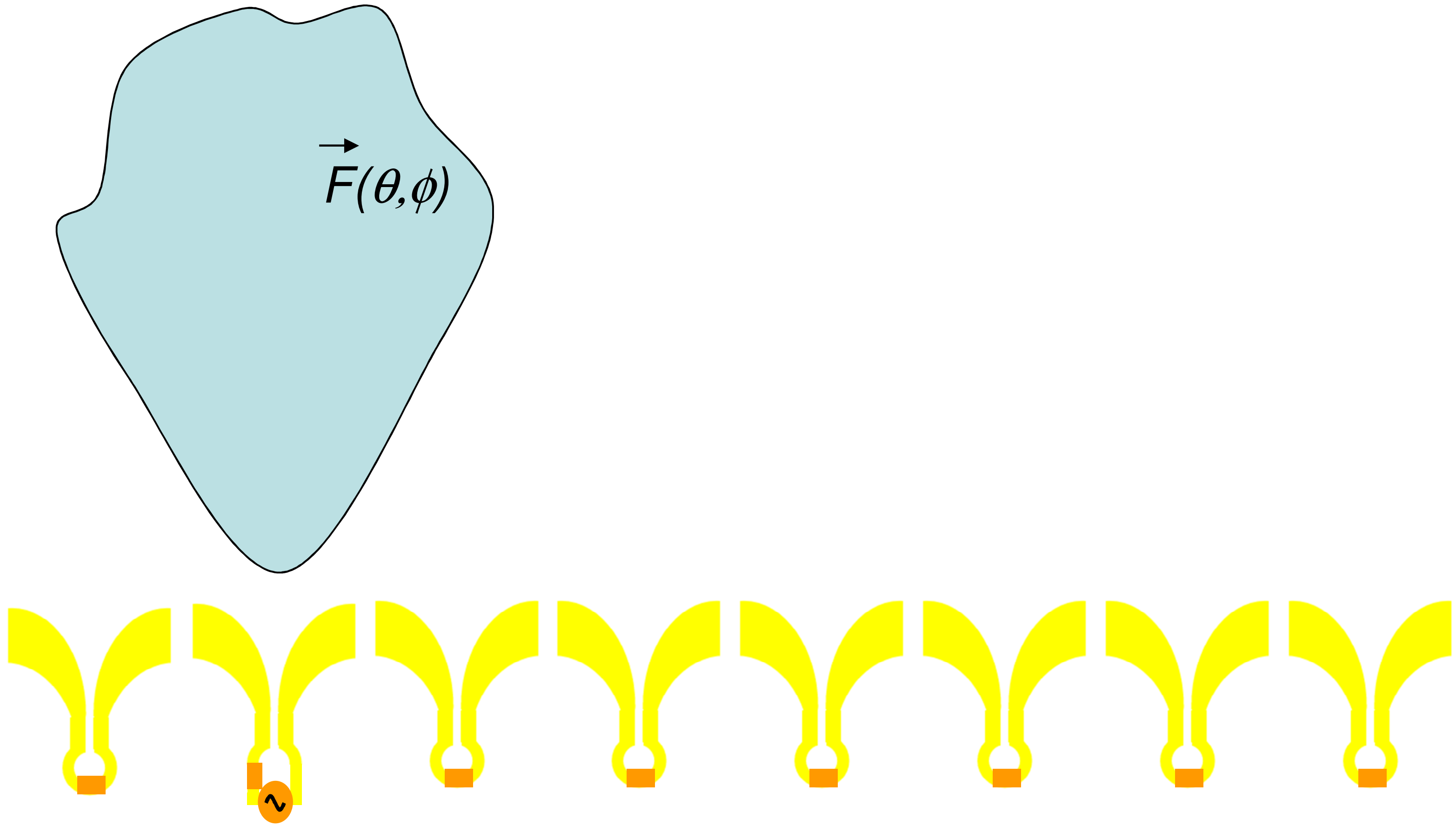

Z_{in}



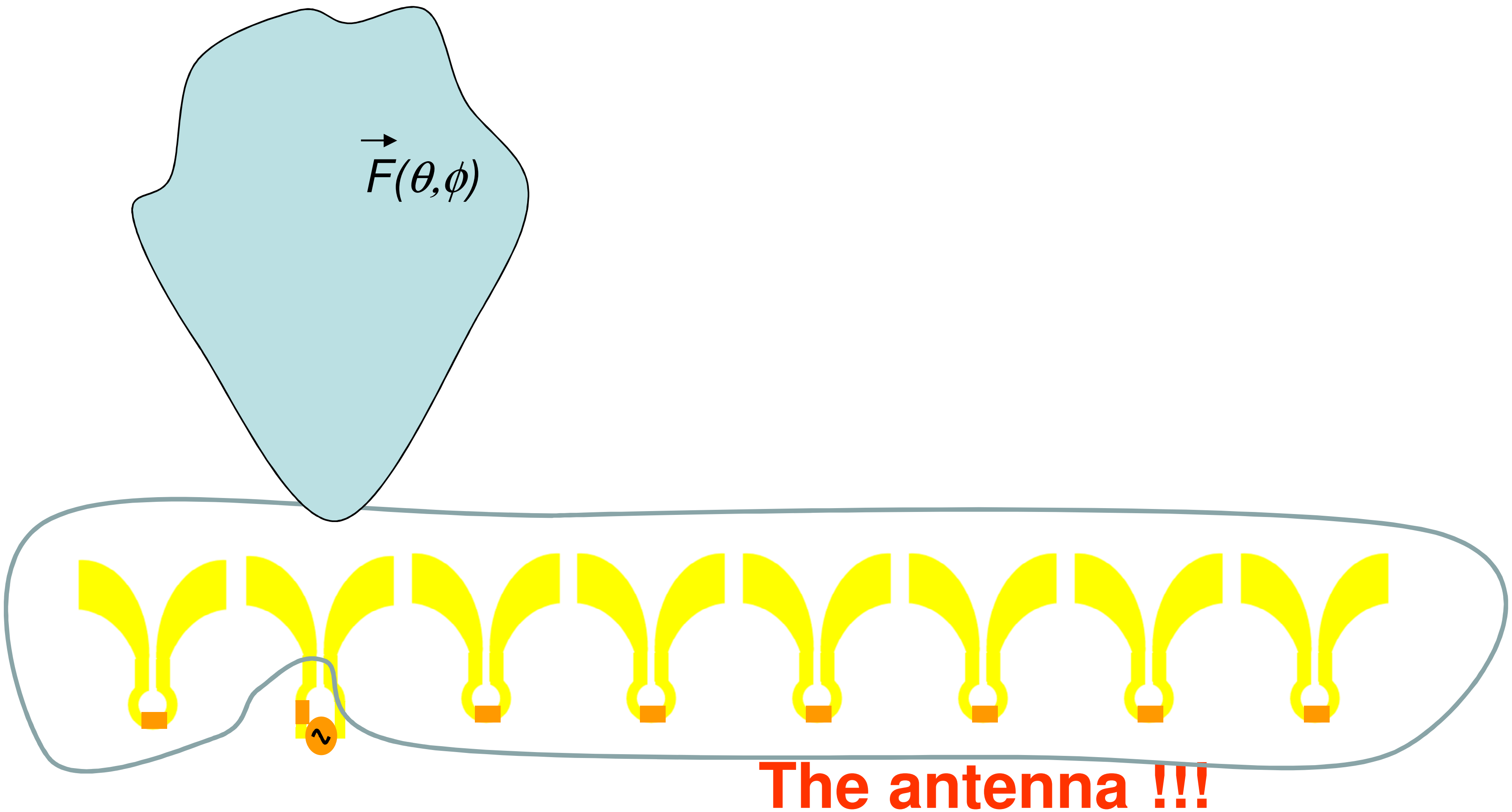

Z'in


The antenna !!!
Z'in

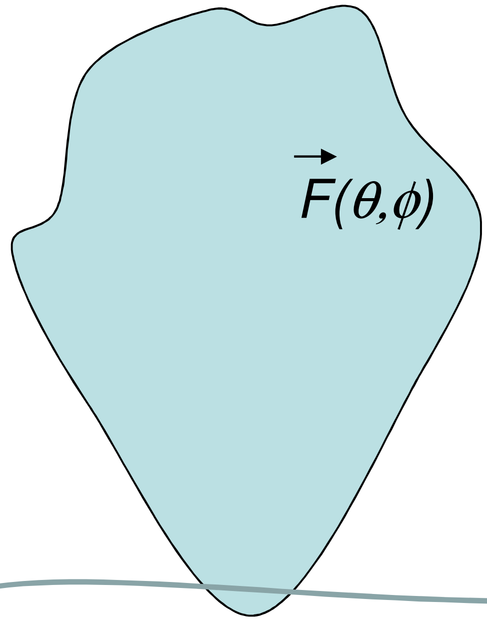
Embedded element pattern



Embedded element pattern



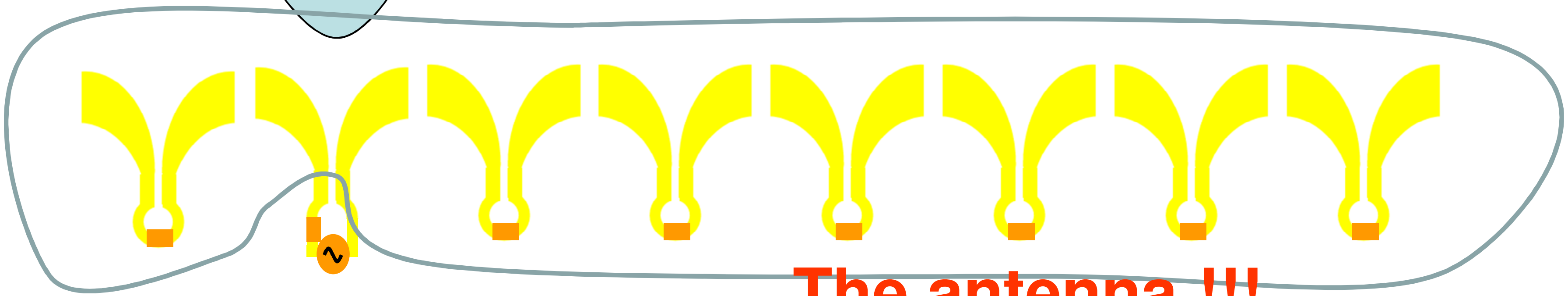
Embedded element pattern



On receive: voltage across load:

$$\frac{\vec{F}(\theta,\phi)\ .\ \vec{E}^{inc}(\theta,\phi)}{j\,k\,\eta}\ \frac{Z_L}{Z_L + Z_p}$$

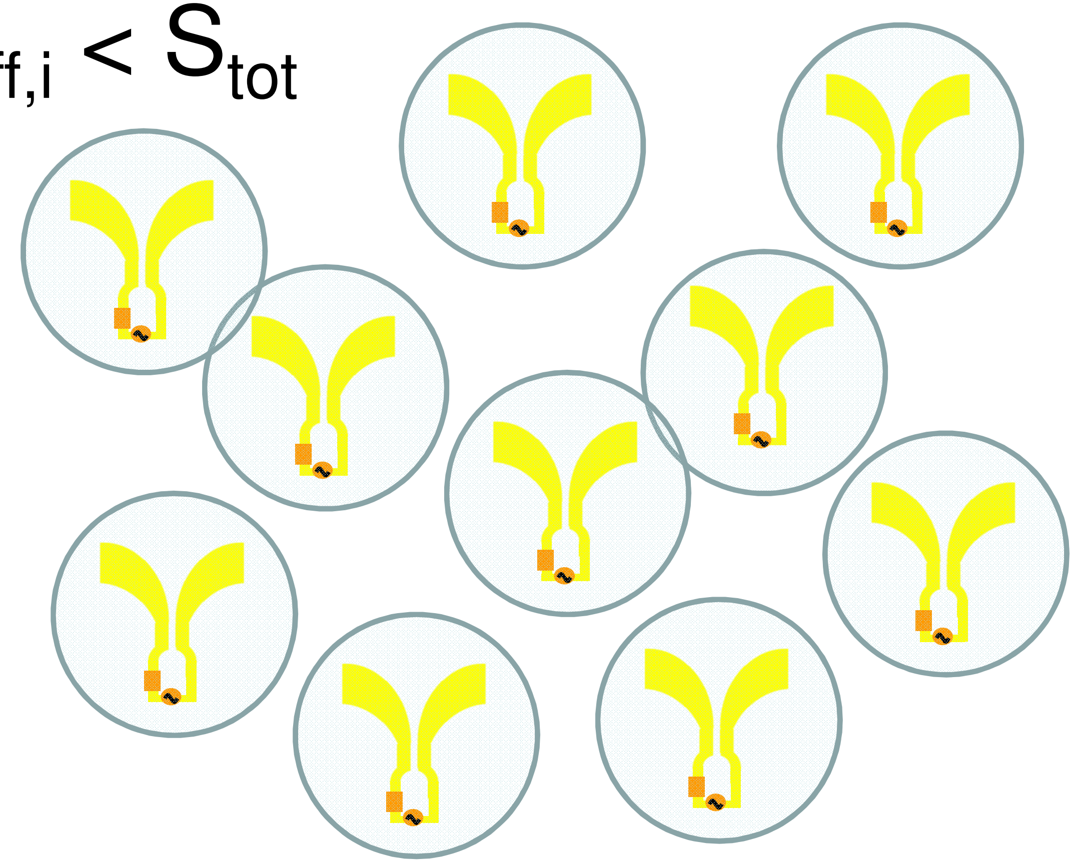
Embedded « passive » impedance



The antenna !!!

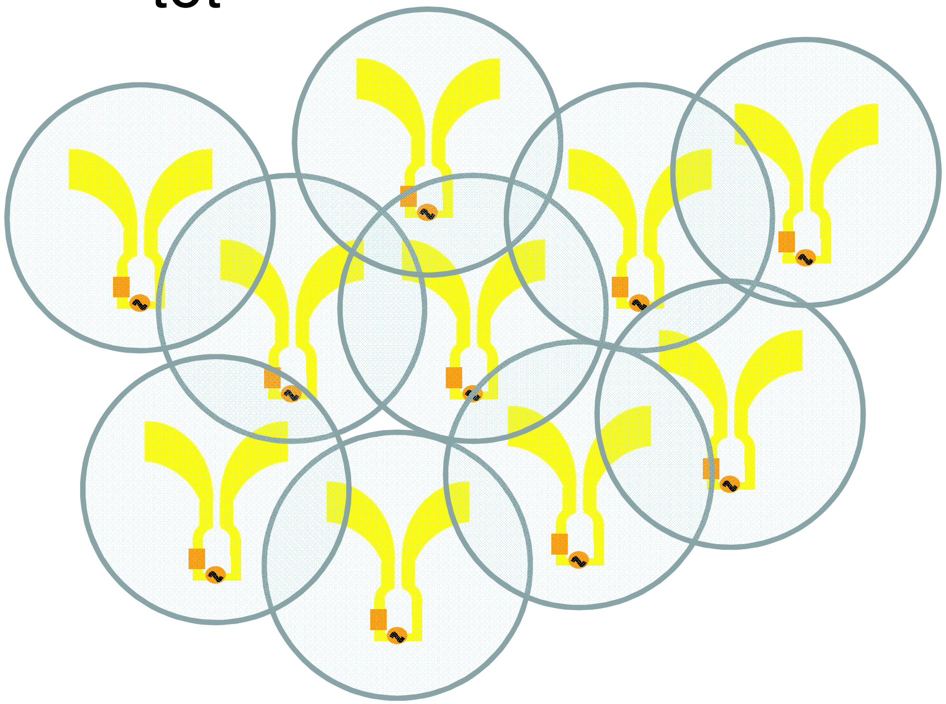
Qualitative approach to MC in dense arrays (1)

$\Sigma \ A_{eff,i} < S_{tot}$



Qualitative approach to MC in dense arrays (2)

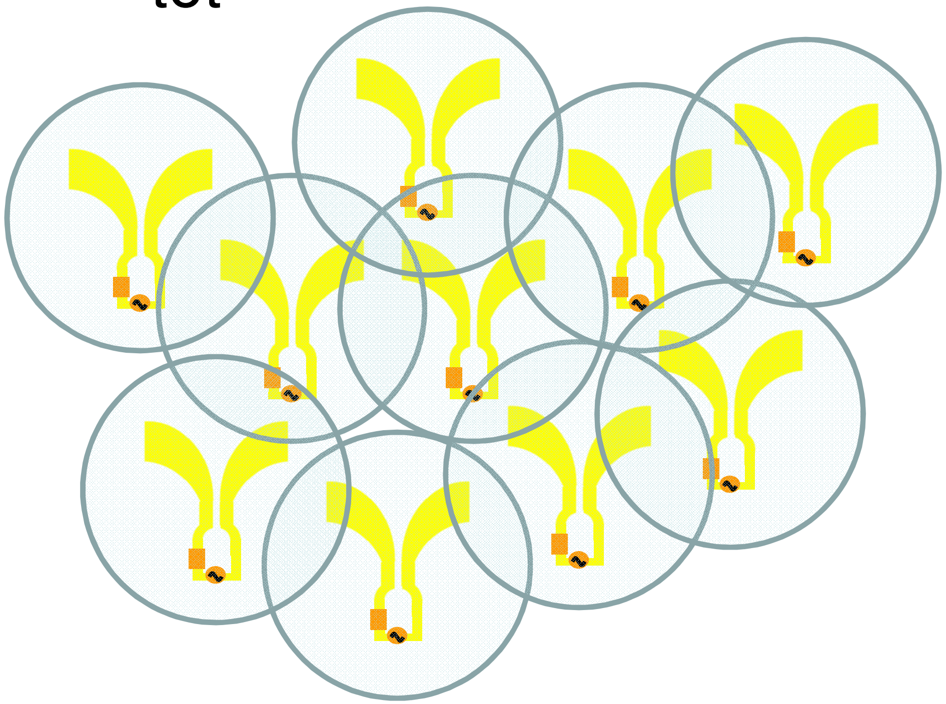
$$\Sigma \; A_{eff,i} > S_{tot}$$



➡ **A_{eff} must shrink** to avoid violation of energy conservation
The embedded patterns must shrink,
e.g. through worse matching, **reduced efficiency**:
on transmit, power **leakage toward loads of neighboring elements**

Qualitative approach to MC in dense arrays (2)

$\Sigma \ A_{eff,i} > S_{tot}$

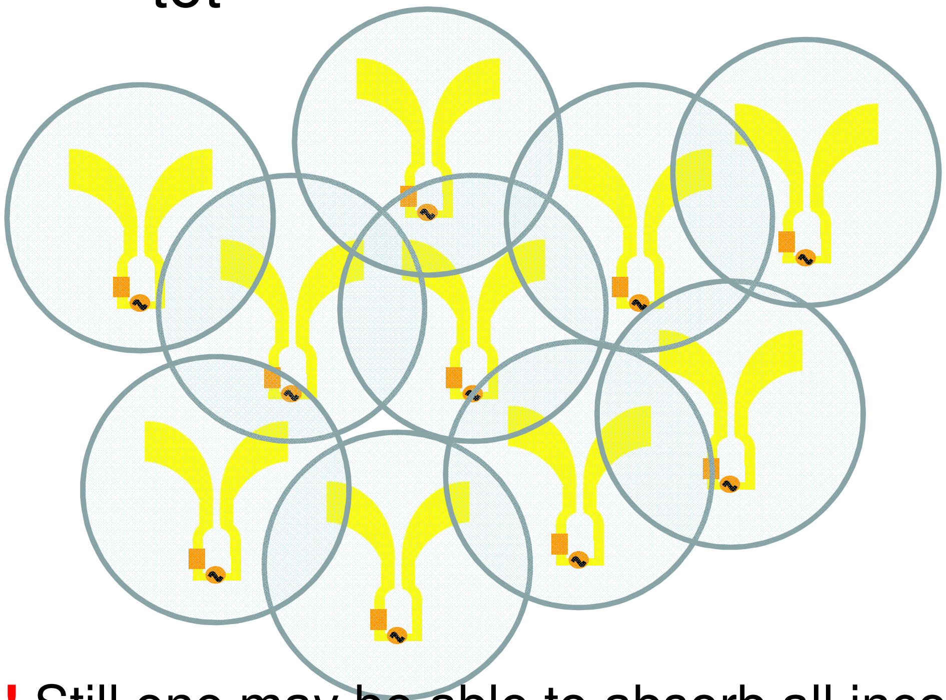


Reduced efficiency, sometimes named « coupling efficiency »

M. Ivashina, M.N. Kehn, P-S Kildal, R. Maaskant, « Decoupling efficiency of a wideband Vivaldi focal plane array feeding a reflector antenna, » IEEE Trans. AP, Feb. 2009.

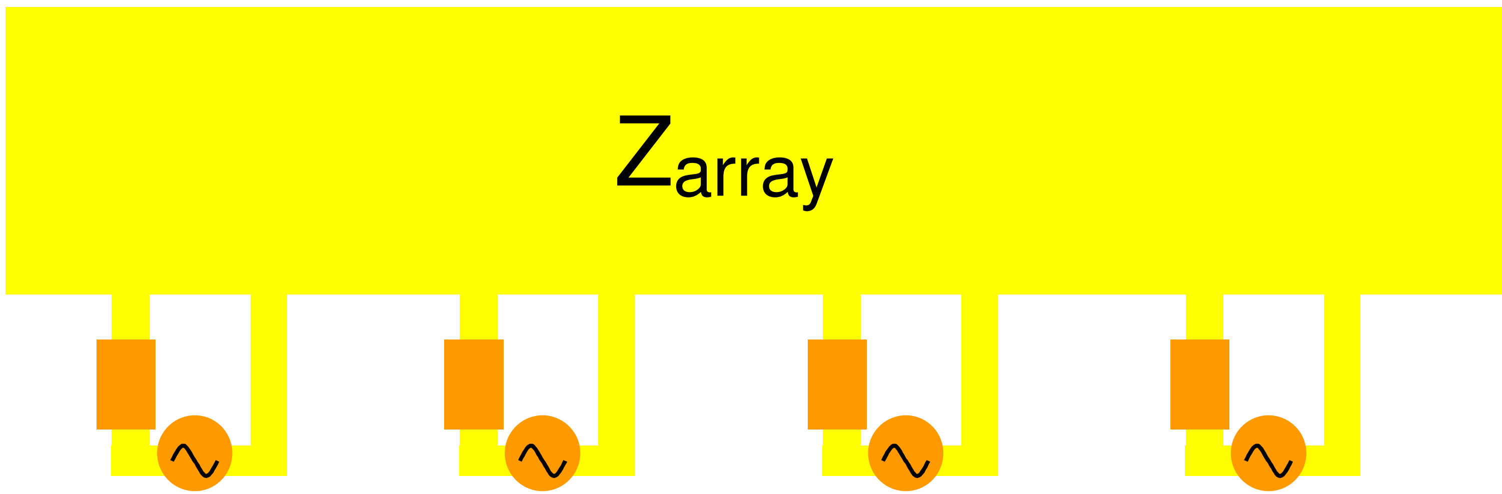
Does that mean lower efficiency on receive ?

$$\Sigma \; A_{eff,i} \sim S_{tot}$$

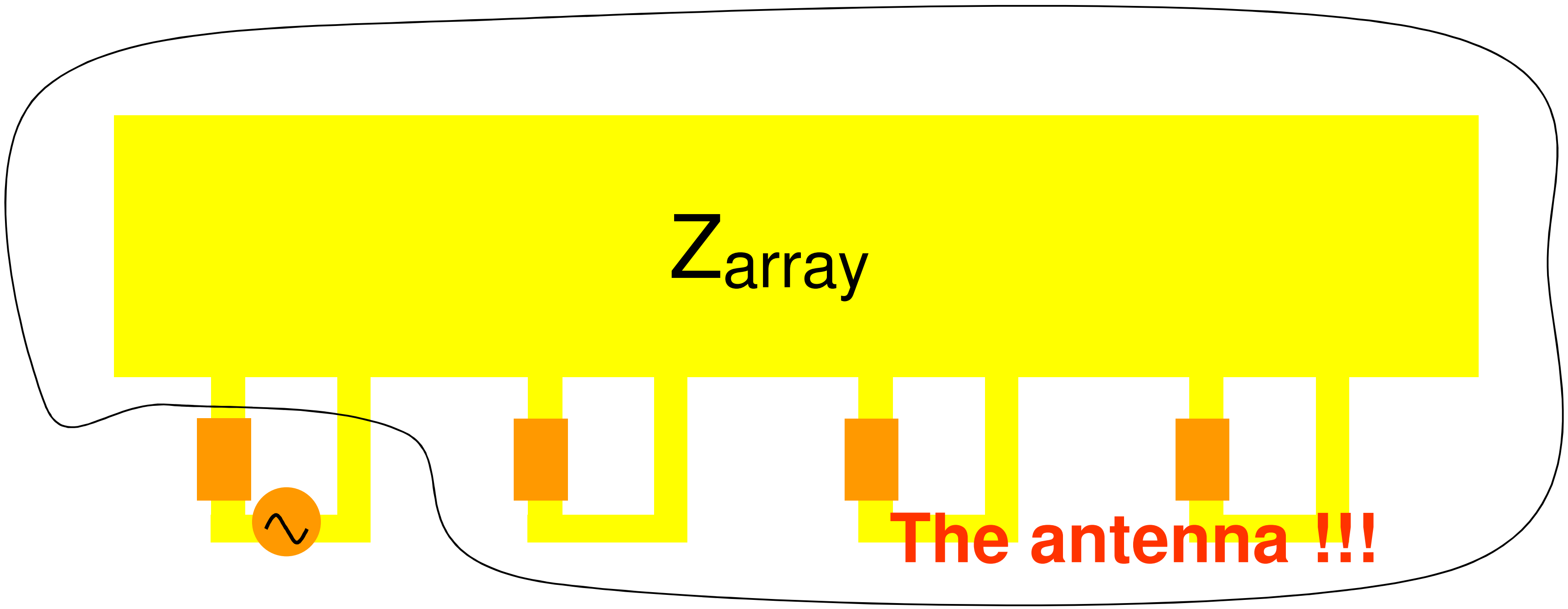


➡ **NO !** Still one may be able to absorb all incoming power (Aeff = Aphys).
Also, on transmit, all the power can be radiated through compensation of what is coupled to a given element by all elements.
cf. active impedance concept (which is excitation-dependent).

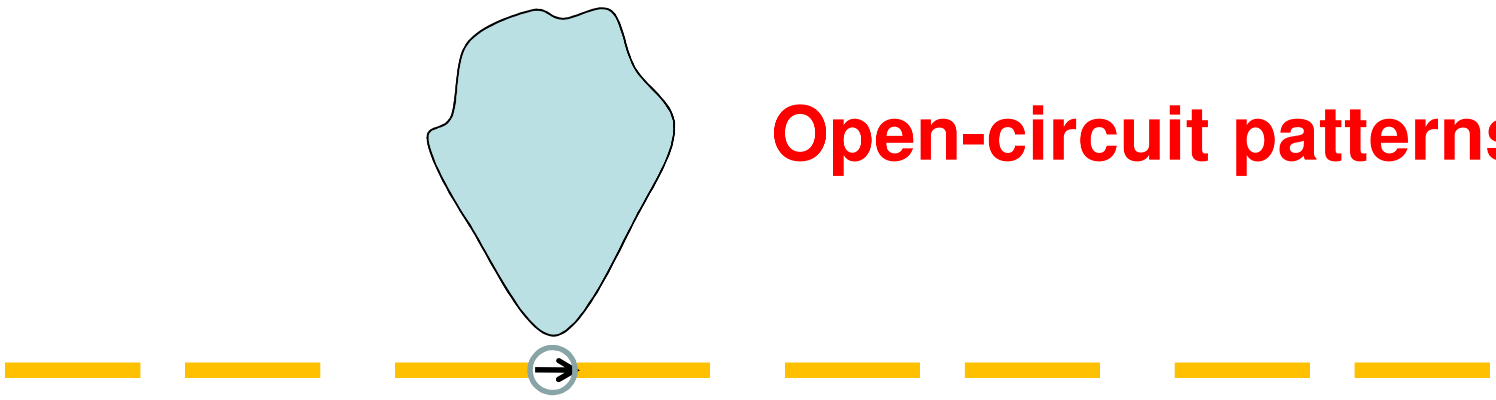
The array from a circuit point of view



The array from a circuit point of view



Open-circuit patterns



o.c. pattern

Embedded pattern

$$\mathbf{g}^{oc} = (\mathbf{Z} + \mathbf{Z}_L)\,\mathbf{g}^{e}$$

In **g**, one column per direction,
one line per antenna.

See S-matrix approach in: K. Warnick and B. Jeffs, “Efficiencies and system temperature for a beamforming array,” IEEE Antennas Propagat. Lett., vol. 7, no. 6, pp. 565–568, Jul. 2008.

Embedded patterns for different sets of loads

$$\left(\mathbf{Z}+\mathbf{Z}_{\mathbf{L},2}\right)\mathbf{g}^{e}_{\mathbf{Z}_{\mathbf{L},2}} = \left(\mathbf{Z}+\mathbf{Z}_{\mathbf{L},1}\right)\mathbf{g}^{e}_{\mathbf{Z}_{\mathbf{L},1}}$$

Minimum-scattering antennas

$$\mathbf{g}^{oc} = (\mathbf{Z} + \mathbf{Z}_L)\, \mathbf{g}^{e}$$

Approximation: o.c. pattern is uncoupled pattern, within constant factor:

$$\mathbf{g}^{oc} \simeq (\mathbf{Z}_d + \mathbf{Z}_L)\, \mathbf{g}^{nc}$$

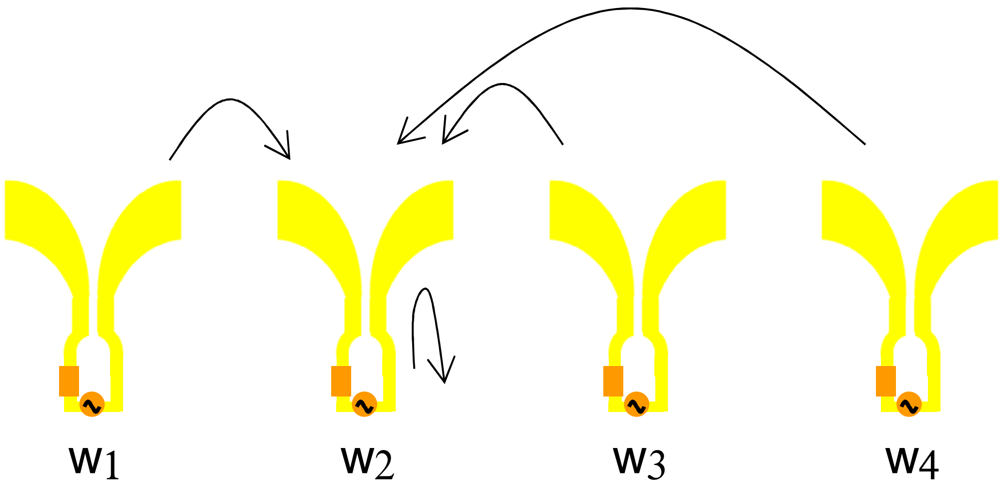
i.e. open antennas ~ invisible

defined with unit current

defined with unit voltage
+ Z_L series impedance

$$\Longrightarrow \mathbf{g}^{e} \simeq (\mathbf{Z} + \mathbf{Z}_L)^{-1}\, (\mathbf{Z}_d + \mathbf{Z}_L)\, \mathbf{g}^{nc}$$

Active impedance : reminder

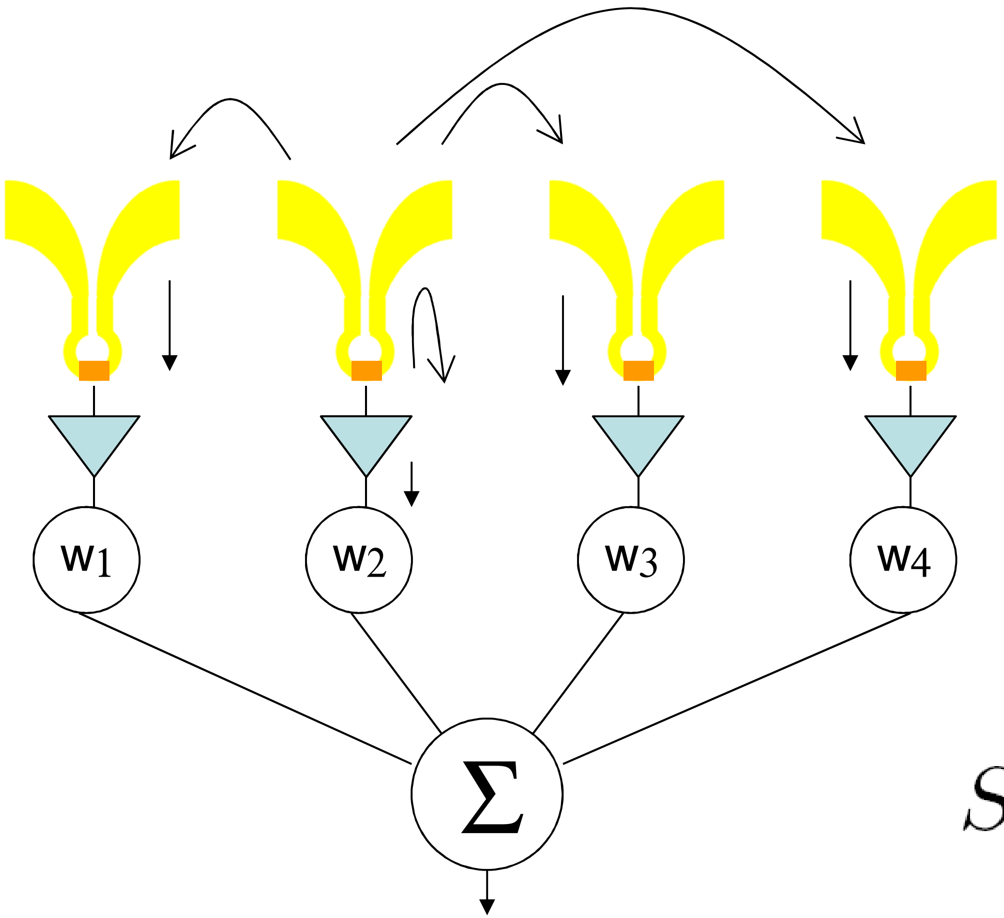


Active reflection coefficient: $$S_{a,i} = \sum_j S_{i,j} \; w_j$$

Active impedance: $$Z_a = Z_\circ (1 - S_a)^{-1} (1 + S_a)$$

K.F. Warnick, M.A. Jensen, « Effects of mutual coupling on interference mitigation with a focal plane array, » IEEE Trans. Antennas Propag, Aug. 2005.

Noise coupling and active impedance



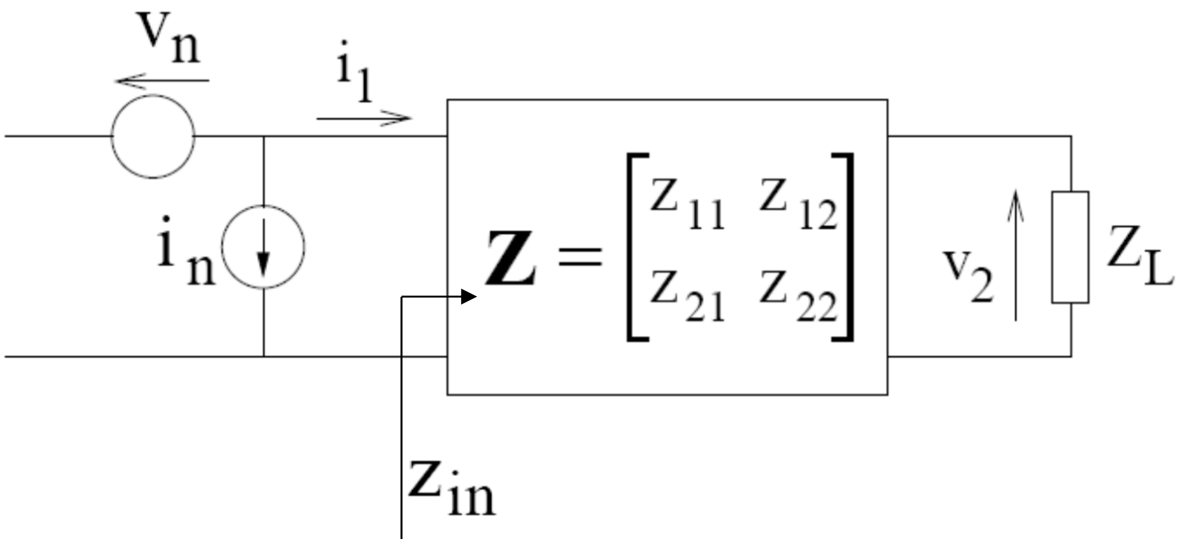
Noise from amplifier i=2 is reflected through active reflection coefficient !

$$S_{a,i} = \sum_j S_{i,j} \; w_j$$

For the effect on total SNR budget:

M. Ivashina, R. Maaskant and B. Woestenburg,,"Equivalent system representation to model the beam sensitivity of receiving antenna arrays, » IEEE AWPL 7, 733–737, 2008.

4-parameter noise model



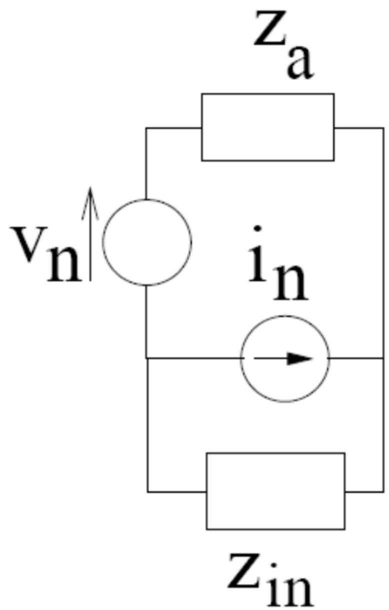
Antenna input impedance

$$\langle |v_n|^2 \rangle = 4\,K\,T_0\,B\,R_n$$

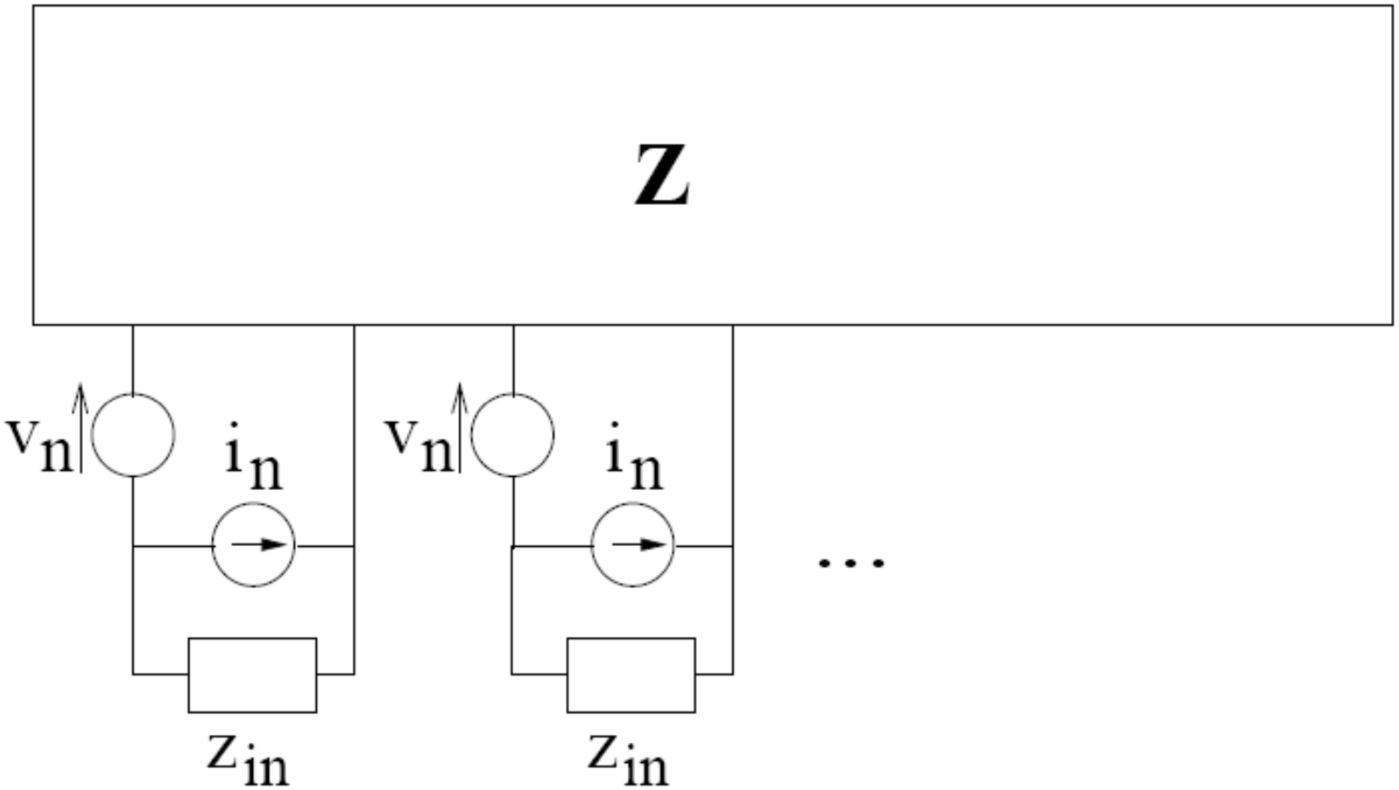
$$\langle |i_n|^2 \rangle = 4\,K\,T_0\,B\,G_n$$

$$\langle v_n\ i_n^\star \rangle = 4\,K\,T_0\,B\,Y_\gamma^\star\,R_n$$

To get the equivalent noise temperature for one antenna

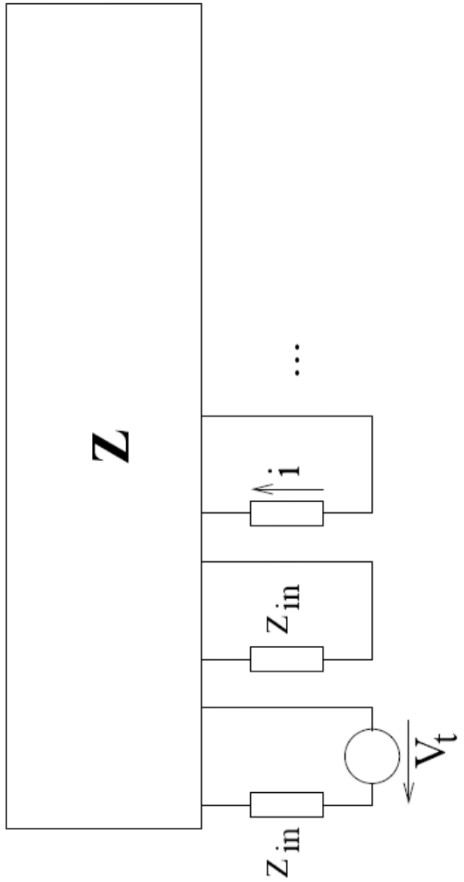


4-parameter noise model (Ctd.)



Craeye, C., B. Parvais, and X. Dardenne, MOM simulation of signal-to-noise patterns in infinite and finite receiving antenna arrays, IEEE Trans. Antennas Propag., 52(12), 3245.3256, Dec. 2004.

Noise vector



Noise voltages on loads:

$$\mathbf{c} = (\mathbf{Z} + z_{in}\,\mathbf{U})^{-1}\,(\mathbf{v_n} + \mathbf{Z}\,\mathbf{i_n})$$

Noise voltage at output of beamformer (one entry per amplifier):

$$\mathbf{c}' = \mathbf{a}\,v_n + \mathbf{b}\,i_n$$

with:

$$\begin{cases} \mathbf{a} = \mathbf{w}^{\mathbf{T}}\,(\mathbf{Z} + z_{in}\,\mathbf{U})^{-1} \\ \mathbf{b} = \mathbf{w}^{\mathbf{T}}\,(\mathbf{Z} + z_{in}\,\mathbf{U})^{-1}\,\mathbf{Z} \end{cases}$$

➡ **Scan-dependent noise** ➡ **SNR pattern**

Unified definitions (Warnick et al. 2010)

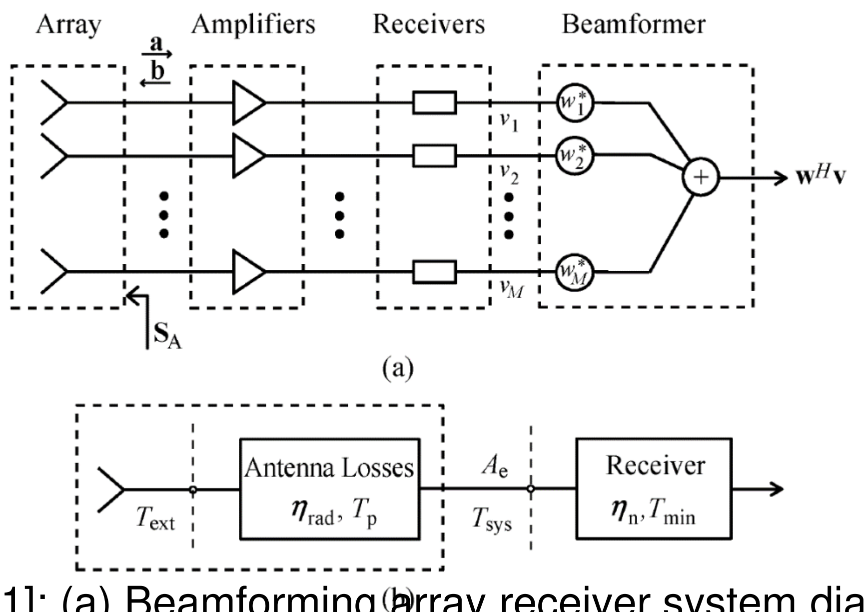


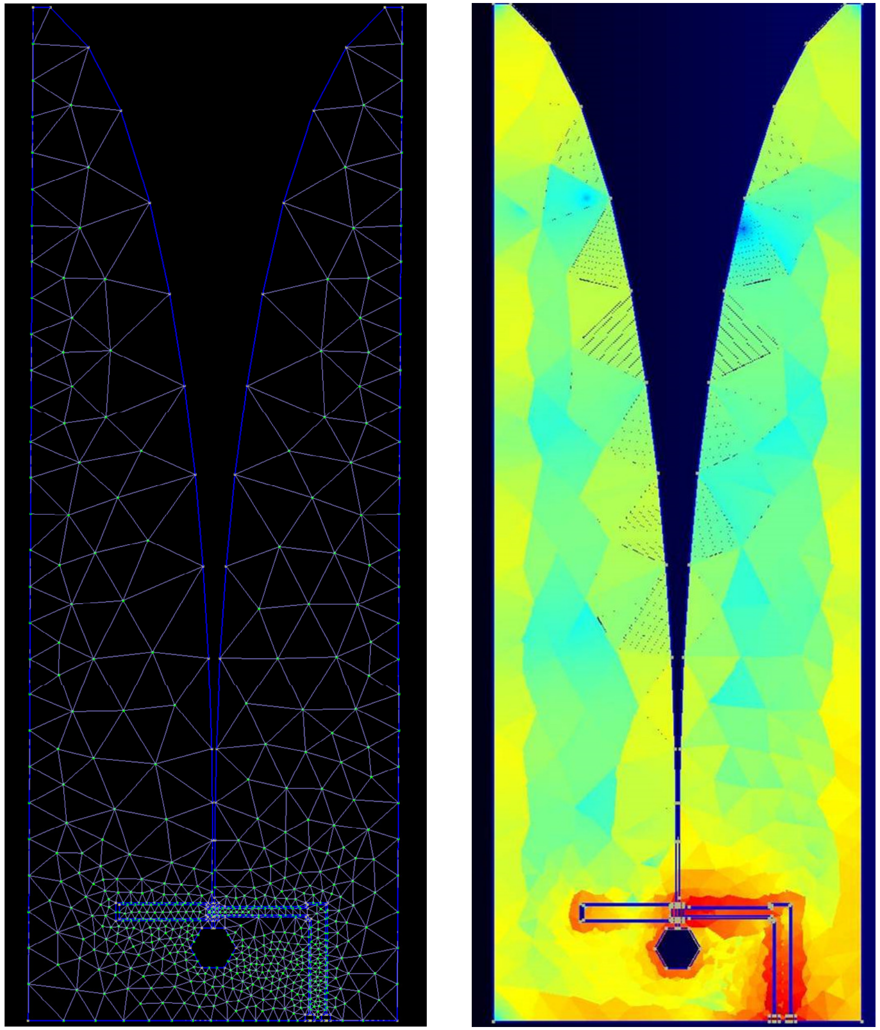
Fig. 1 of [1]: (a) Beamforming array receiver system diagram.
(b) Equivalent system with reference planes indicated.

Establishment of definitions regarding noise temperature, efficiencies, gain… for arrays that match the IEEE definitions for isolated antennas.

[1] K.F. Warnick, M. V. Ivashina, R. Maaskant, and B. Woestenburg, « Unified definitions of efficiencies and system noise temperature for receiving antenna arrays," *IEEE Trans. Antennas Propag., Vol.* 58(6), 2121–2125.

[2] M. Ivashina, R. Maaskant and B. Woestenburg,,"Equivalent system representation to model the beam sensitivity of receiving antenna arrays, » IEEE AWPL 7, 733–737, 2008.

Method-of-Moment analysis of antenna arrays



Tapered-slot antenna with complex balun

- 1402 unknowns on the metallic antenna
- 2 x 380 unknowns on the dielectric box
- 16 X 16 array

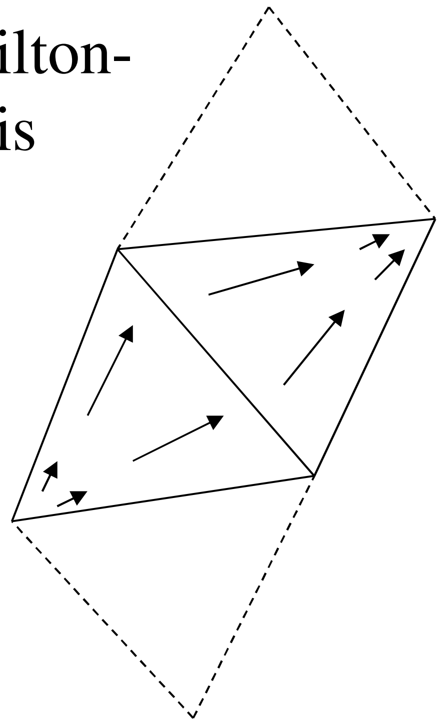
553.472 unknowns

Simulations by Hanni Sarafin, design from Bruce Veidt et al.

Basic principle of Method of Moments

Discretization: $$\vec{J}(\vec{r}\,') = \sum_{i=0}^{N} x_i \, \vec{J}_{b,i}(\vec{r}\,')$$

The Rao-Wilton-Glisson basis function



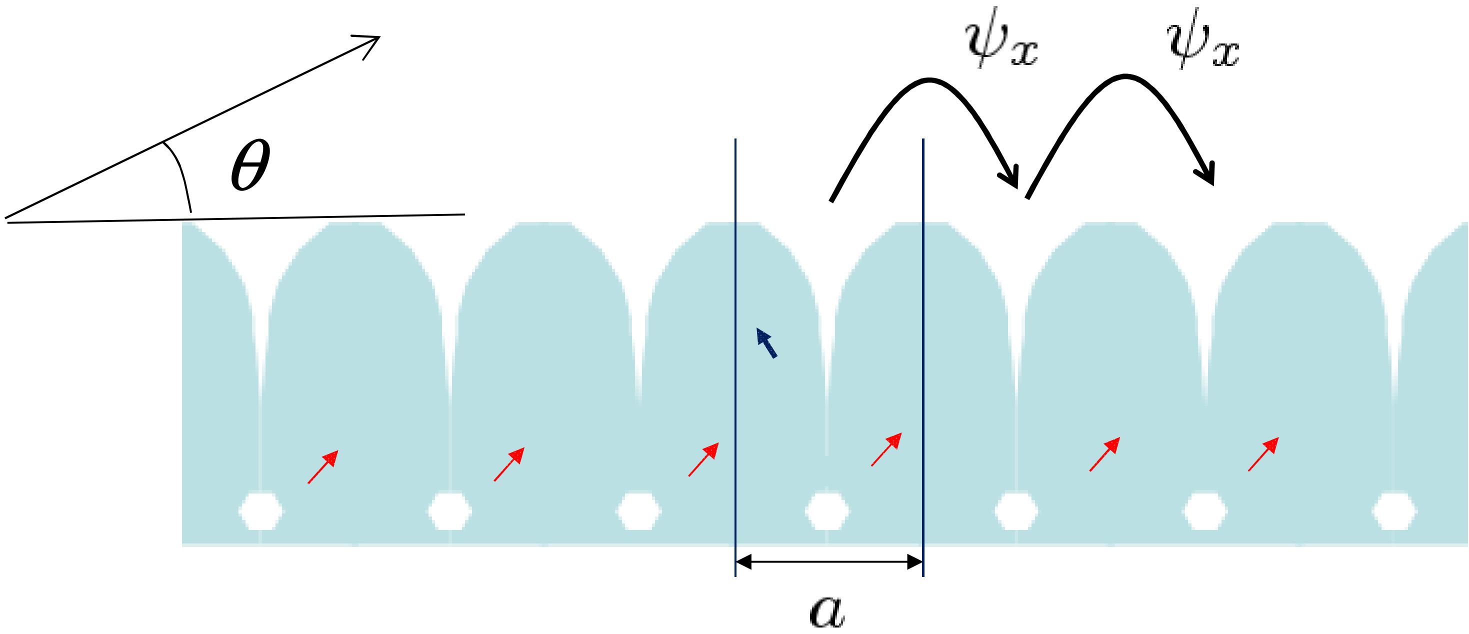
Obtain x_i by ensuring **boundary conditions** everywhere on surface.

For PEC, tangential electric field is zero.

In practice ensure **on N points** OR in a locally average sense, on the domain of **testing** functions.

N equations with N unknowns.

Numerical analysis of infinite arrays



$$\psi_x = k_0\, a \, \cos\theta$$

Impose boundary conditions in unit cell based on fields radiated by basis functions located in ALL cells, with appropriate phase shift

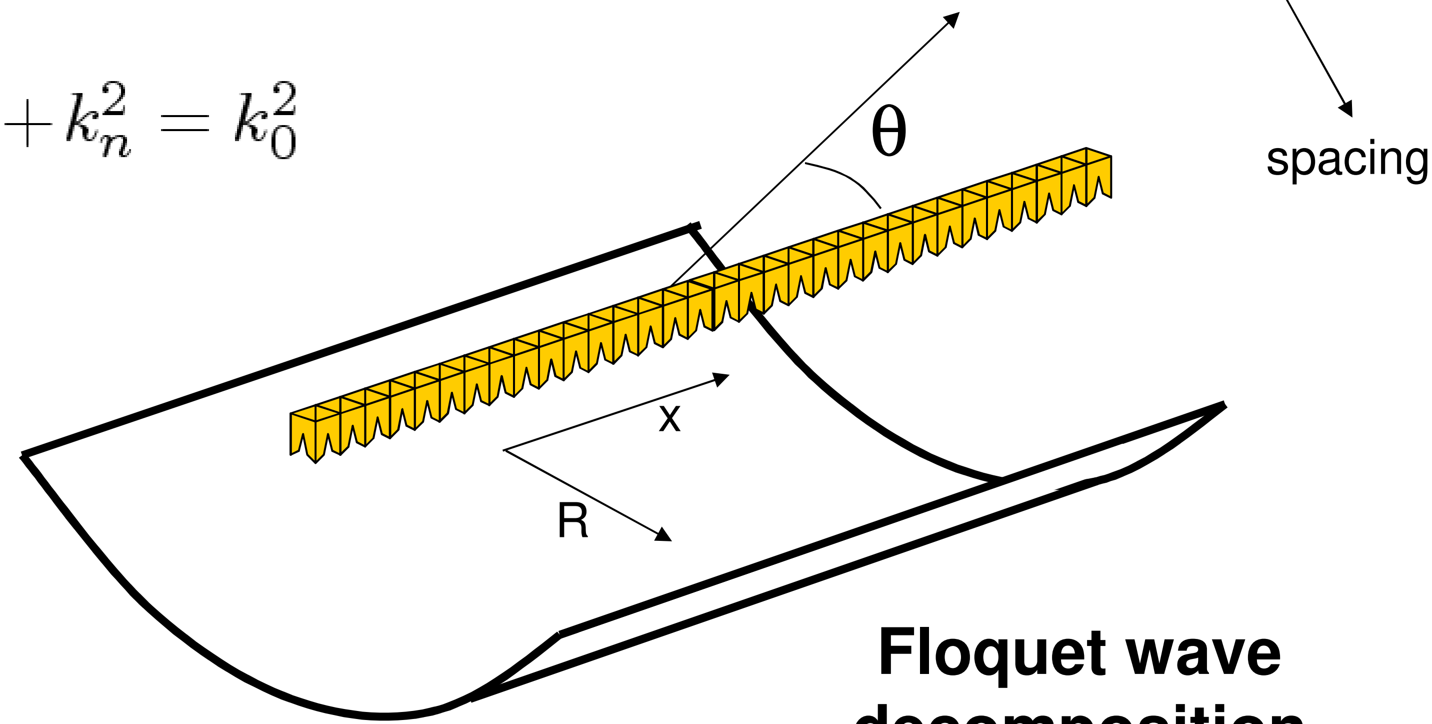
Periodic Green's functions

$$G(x,R) = \frac{1}{4\,j\,a} \sum_{n=-\infty}^{\infty} H_0^{(2)}(\gamma_n\, R)\, e^{-j\,k_n\,x}$$

$$\psi_x = k_0\, a\, \cos\theta$$

$$k_n = \psi_x/a + n\,2\,\pi/a$$

$$\gamma^2 + k_n^2 = k_0^2$$



Floquet wave decomposition

Array Scanning Method with finite resolution

$$\underbrace{I(m)}_{\text{Current at ant. m for ant. 0 excited}} = \frac{1}{2\pi} \int_0^{2\pi} \underbrace{I^{\infty}(\psi)}_{\text{Infinite-array solution for phase shift } \psi \text{ between elements}} e^{-j\,m\,\psi}\, d\psi$$

Current at ant. m
for ant. 0 excited

Infinite-array solution for phase
shift ψ between elements

$$I(m) \simeq \frac{1}{N} \sum_{p=0}^{N-1} I^{\infty}(\psi_p)\, e^{-j\,m\,\psi_p}$$

$$\psi_p = 2\,\pi\,p/N$$

Aliasing:

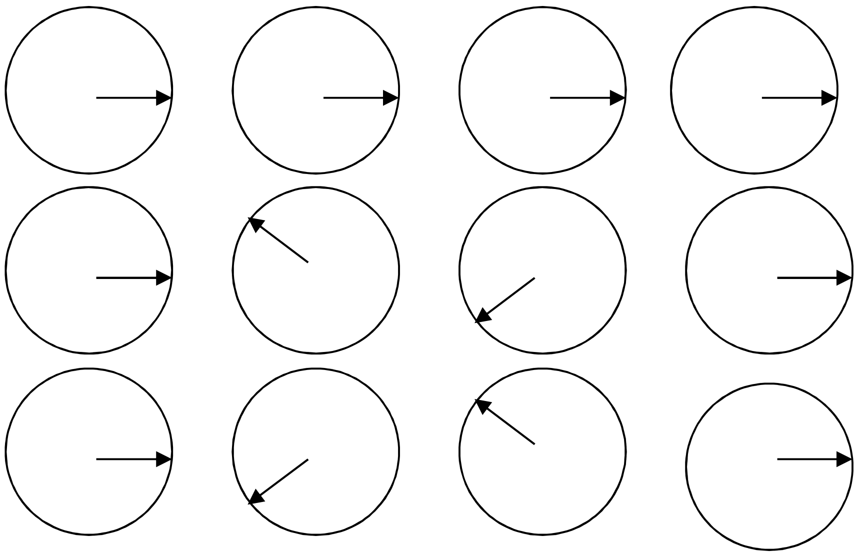
Repetition of source
every N elements

B.A. Munk and G.A. Burrell, « plane-wave expansion for arrays of arbitrarily oriented piecewise linear elements and its application in determining the impedance of a single linear antenna in a lossy half-space, » IEEE Trans. AP, vol 27, pp. 331-343, 1979.

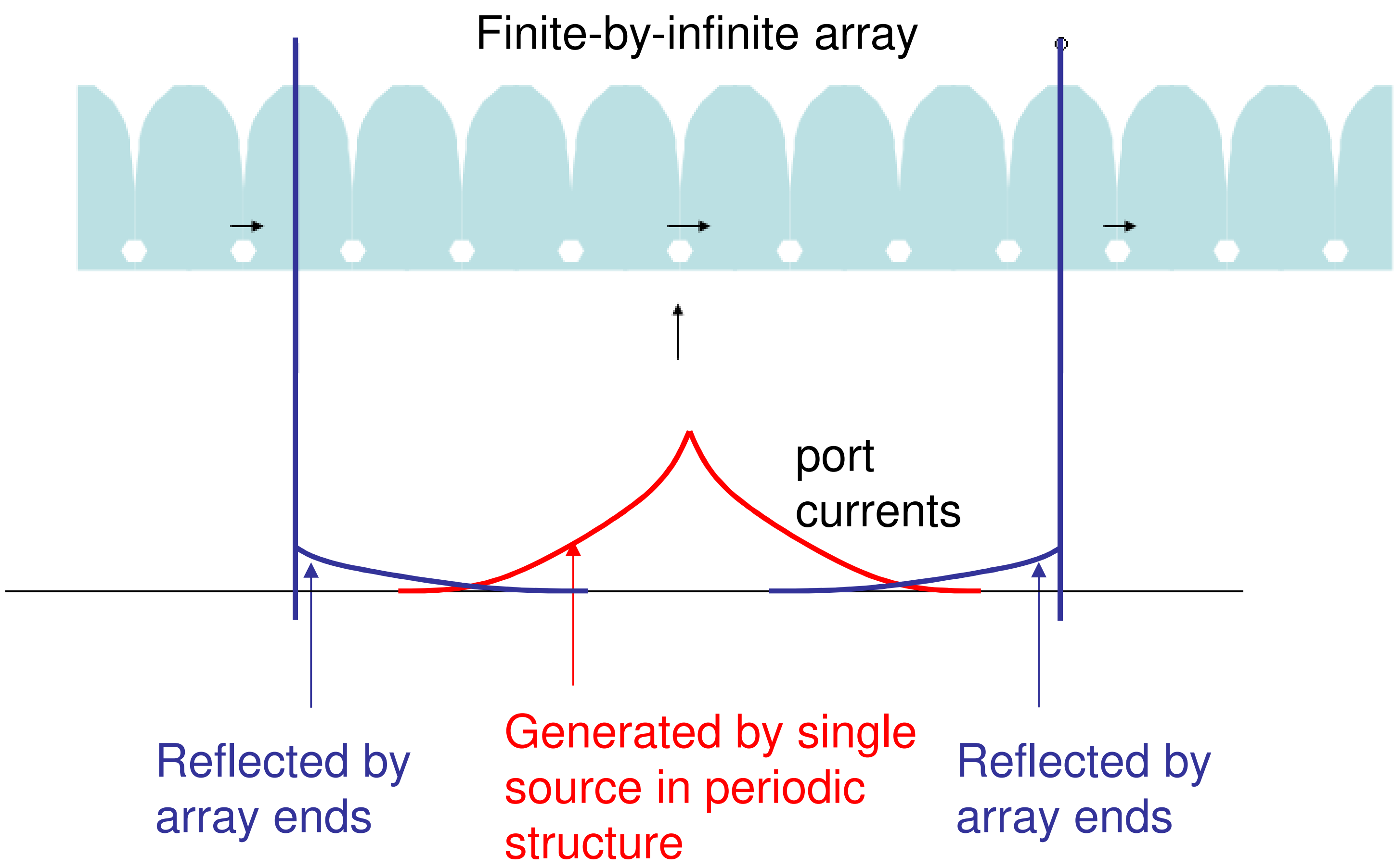
Array Scanning Method with finite resolution

$$I(m) \simeq \frac{1}{N} \sum_{p=0}^{N-1} I^{\infty}(\psi_p)\, e^{-j\, m\, \psi_p}$$

1	1	1	1	1	1	1
1	-1	1	-1	1	-1	1
2	0	2	0	2	0	2

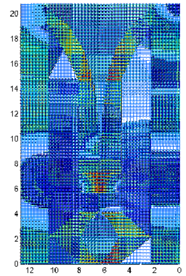


Wave phenomenology in finite arrays

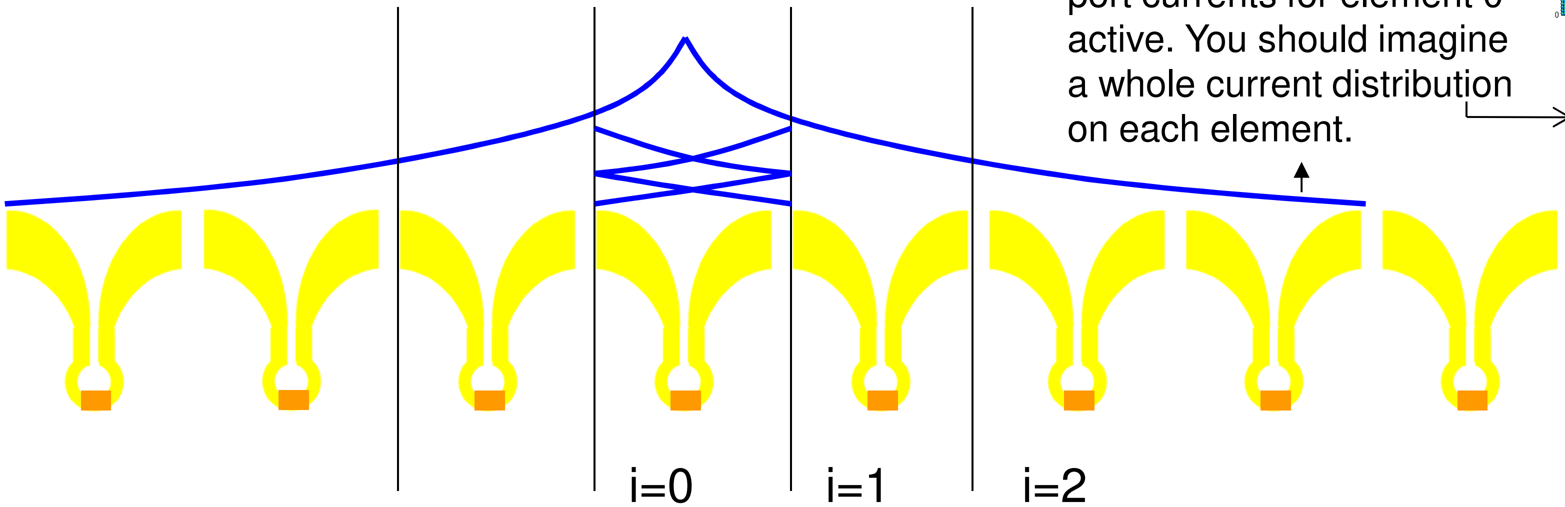


~ same distributions as forward wave, except for edge elements

Embedded pattern in infinite array



NB: this curve represents port currents for element 0 active. You should imagine a whole current distribution on each element.



$$E_p = C \sum_i \iint_S \vec{J}_i \, . \, \hat{e}_p \; e^{j \, \vec{k} \, . \, \vec{r}\,'} \; dS$$

o

Correct for phase shifts due to « wrong » position

$$C = -j \, \omega \, \mu \, e^{-j \, k \, R} / (4 \pi \, R)$$

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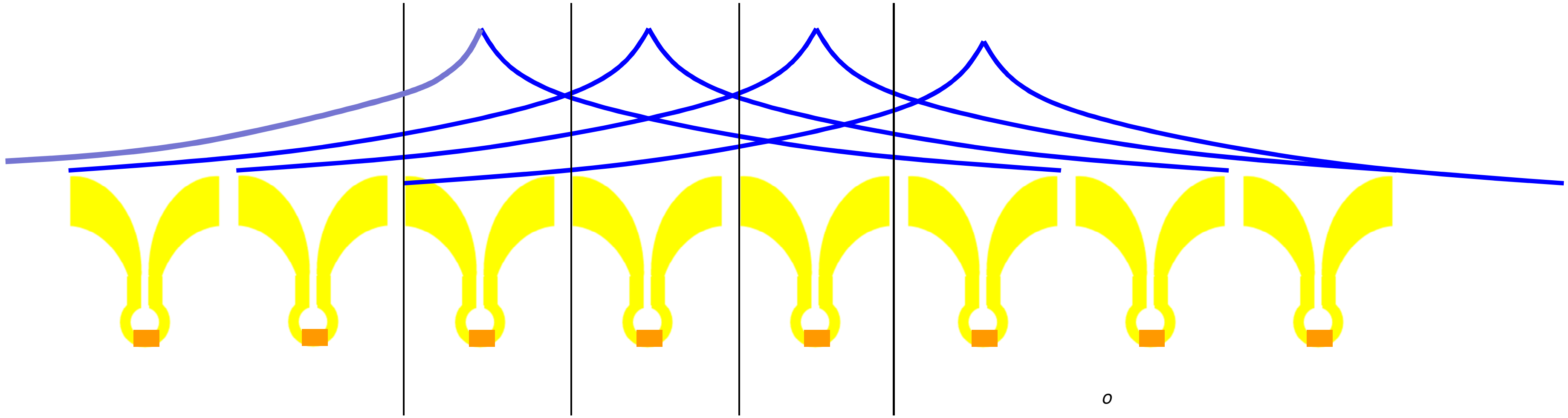
Embedded pattern in infinite array

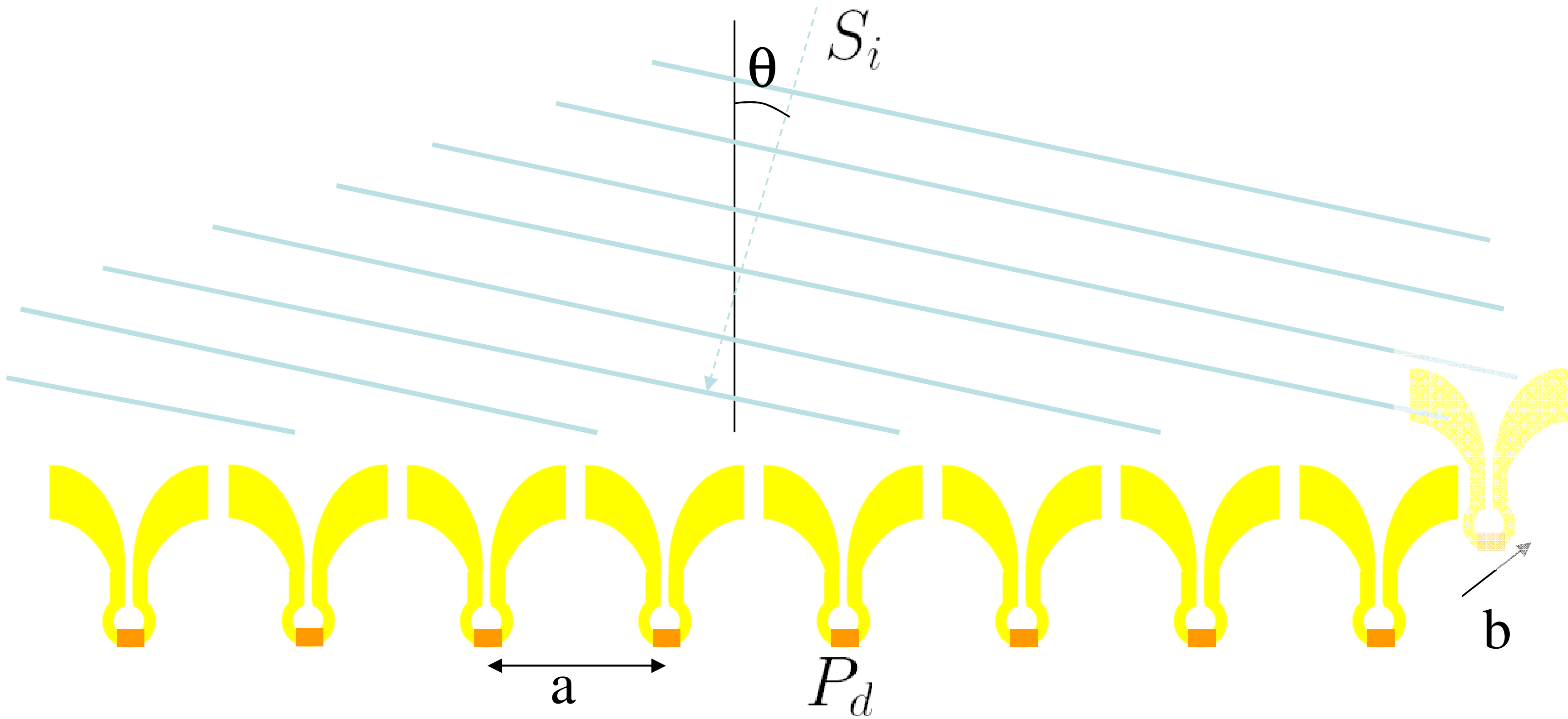


$$E_p = -j\,\omega\,\mu\,\frac{e^{-j\,k\,R}}{4\pi\,R} \iint_{S_\circ} \left(\sum_i \vec{J}_i \,.\, \hat{e}_p\, e^{j\,\psi\,i} \right) e^{j\,\vec{k}\,.\,\vec{r}'_\circ}\; dS_\circ$$

Infinite-array current

Conclusion: in infinite arrays, the « embedded element pattern » is simply obtained from infinite-array results. Therefore, it is sometimes also called « active element pattern ». The infinite array must be scanned toward the direction of interest !

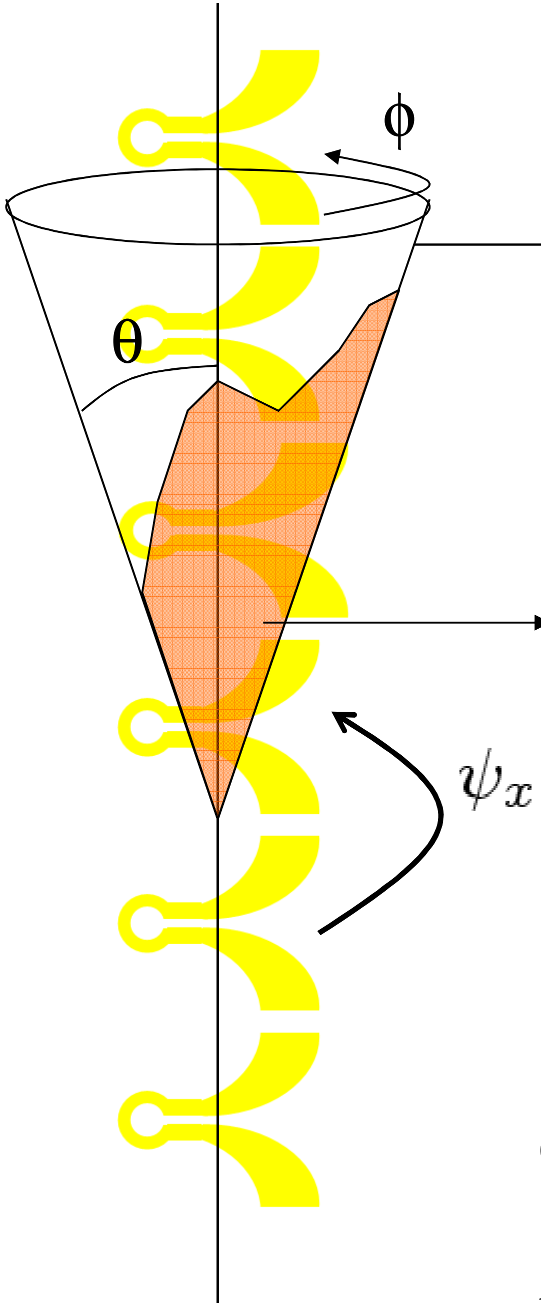
Receiving cross-section: $A(\hat{u}, \vec{e}_i) = \frac{P_d}{S_i}$



Planar array: $A_r(\theta, \phi) = \cos\theta \ \ a\,b \left(1 - |\Gamma_a|^2\right) \eta_{pol}$

Active reflection coefficient

Directivity



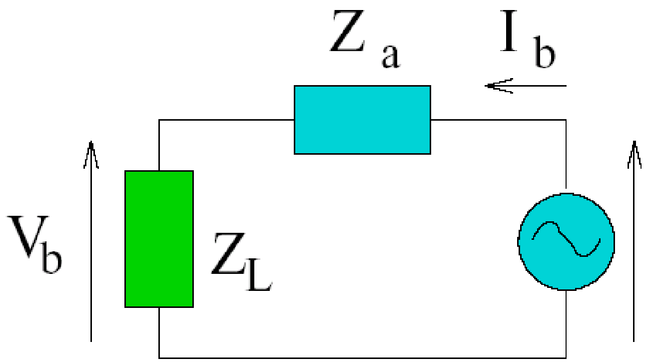
Radiation within cone

Specific pattern within cone

$$D_\theta(\phi) = \frac{|\vec{E}_a|^2/(2\,\eta)\ \cos\theta\, 2\,\pi\, R\, a}{\mathcal{R}\{Z_A\}|I_a|^2/2}$$

C. Craeye, M. Arts, "On the receiving cross section of an antenna in infinite linear and planar arrays," Radio Science, April 2004.

Receiving cross-section for linear array



$$V_o = \frac{E^\circ \; E_a^\circ}{\omega \, \mu} \, \hat{n}_a \, . \, \hat{n}_b \;\; 4 \, a \cos \phi_{pol}$$

$$P_d \;=\; 1/2 \, \mathcal{R} \left\{ \left| \frac{Z_L}{Z_A + Z_L} V^\circ \right|^2 \frac{1}{Z_L^\star} \right\}$$

$$A(\hat{u}, \vec{e}_i) = \frac{P_d}{S_i}$$

$$A_r(\theta, \phi) = D_\theta(\phi) \, \frac{\lambda \, a}{2 \, \pi} \left(1 - |\Gamma_a|^2\right) \, \eta_{pol}$$

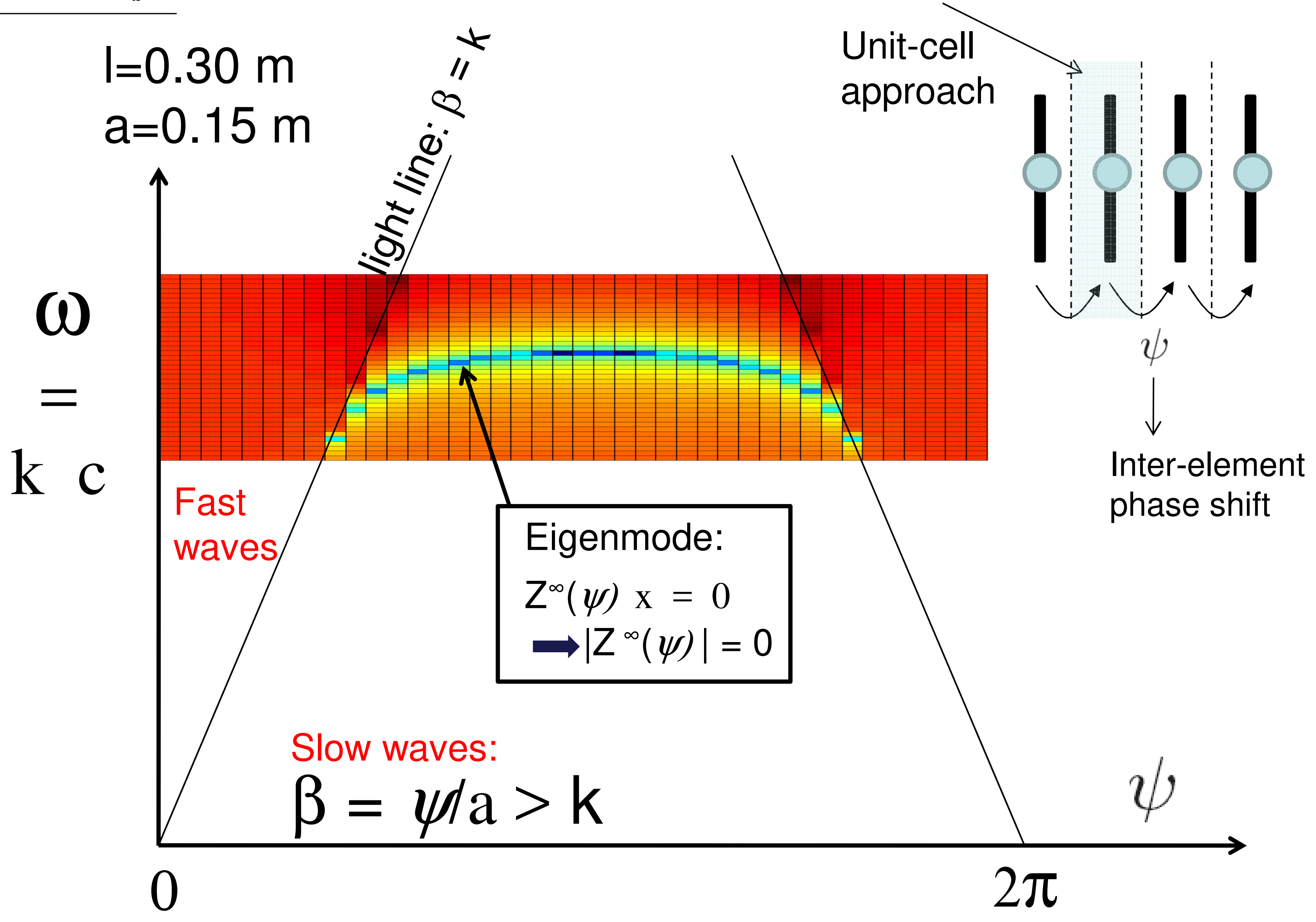
Receiving cross-section in 3 different cases

Isolated	λ^2	1	$D(\theta,\phi)$	$1/4\pi$	$(1-\|\Gamma\|^2)$	$\|\cos\phi_p\|^2$
Linear	λ	a	$D_\theta(\phi)$	$1/2\pi$	$(1-\|\Gamma_a(\theta)\|^2)$	$\|\cos\phi_p\|^2$
planar	1	ab	$\cos\theta$	1	$(1-\|\Gamma_a(\theta,\phi)\|^2)$	$\|\cos\phi_p\|^2$

Linear array appears as a quite intuitive transition between isolated element and planar array cases

C. Craeye, M. Arts, "On the receiving cross section of an antenna in infinite linear and planar arrays," Radio Science, April 2004.

Dispersion curves (from infinite-array simulations)



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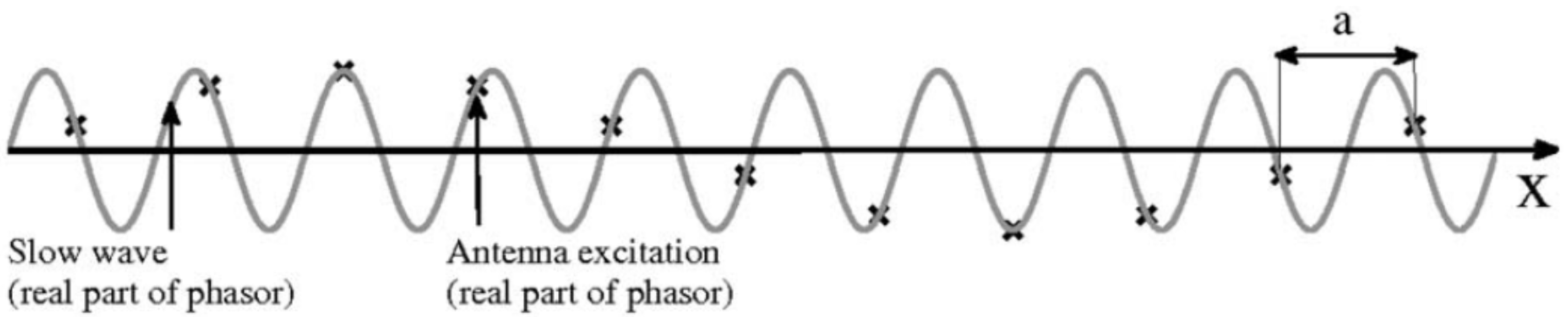
Scan blindness: aliasing on slow wave



The antennas collectively excite a fast wave (visible region) BUT, for a specific scan angle, they are in phase with the slow wave excited by each element

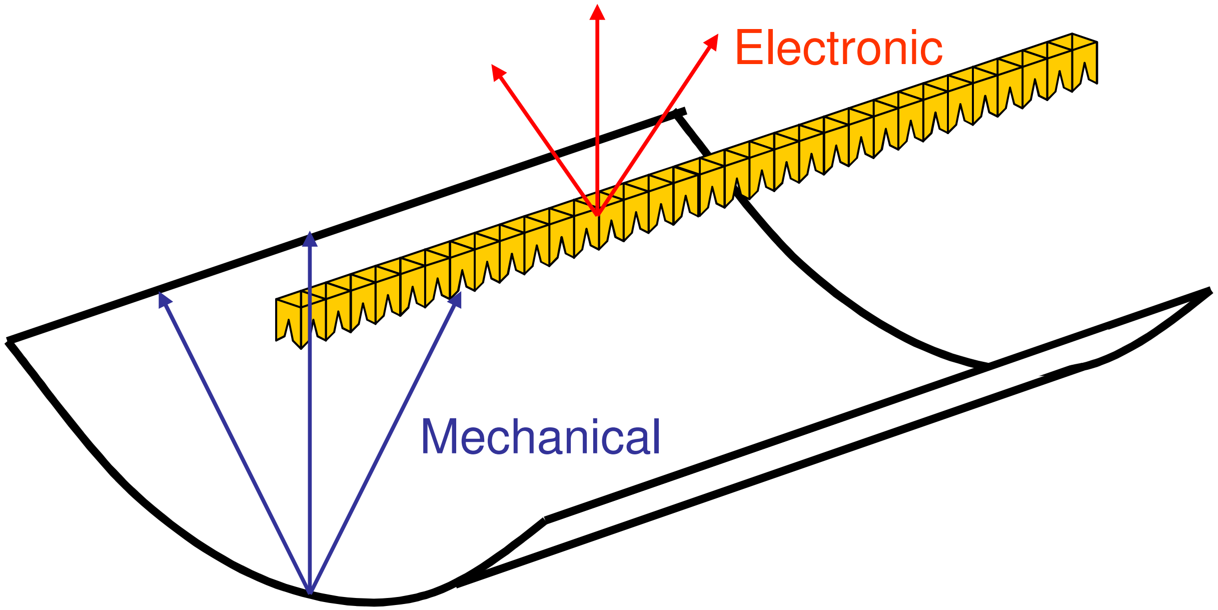
$\Rightarrow$ All the power goes to the slow wave, no radiation, purely imaginary active impedance

C. Craeye, D. Gonzalez-Ovejero, « A review on array mutual coupling analysis, » Radio Science, April 2011.

Cylindrical reflectors

Mix of mechanical and electronic scanning.

Trade-off between cost/complexity and adaptive capabilities



Radar systems

New generation of radio telescopes

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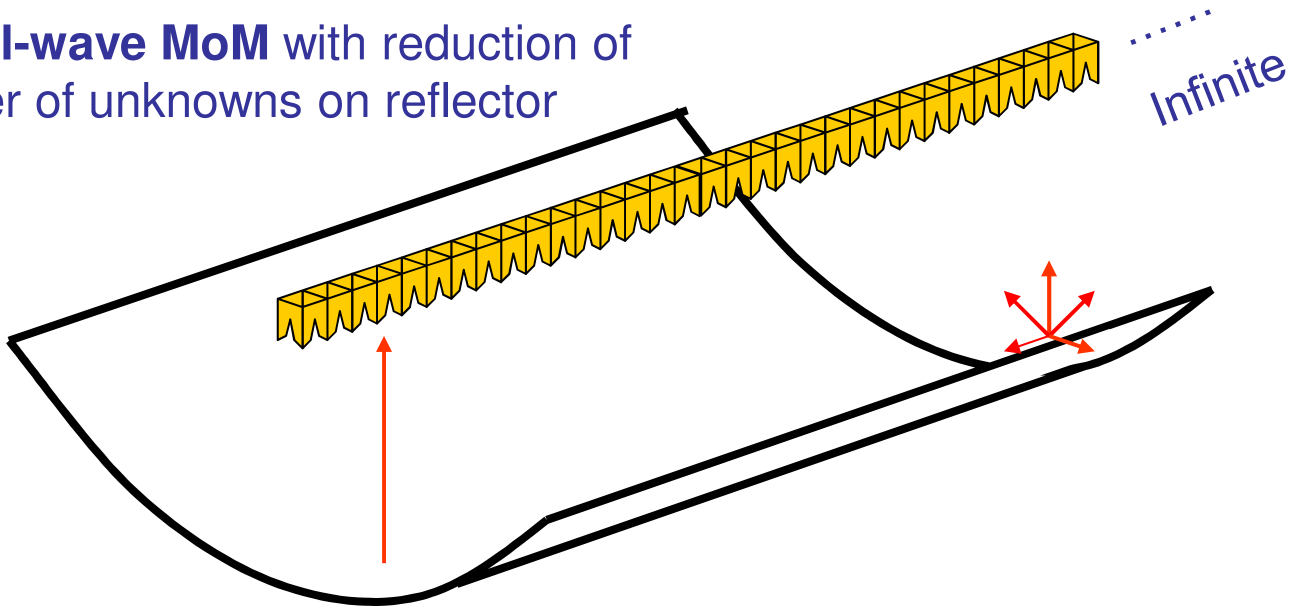
Array-reflector interaction

(a) primary element patterns of antennas in the array,
(b) secondary pattern obtained after reflection.

Step (b): physical optics + diffraction by edges + blockage
=> not so easy !

=> **Full-wave MoM** with reduction of
number of unknowns on reflector



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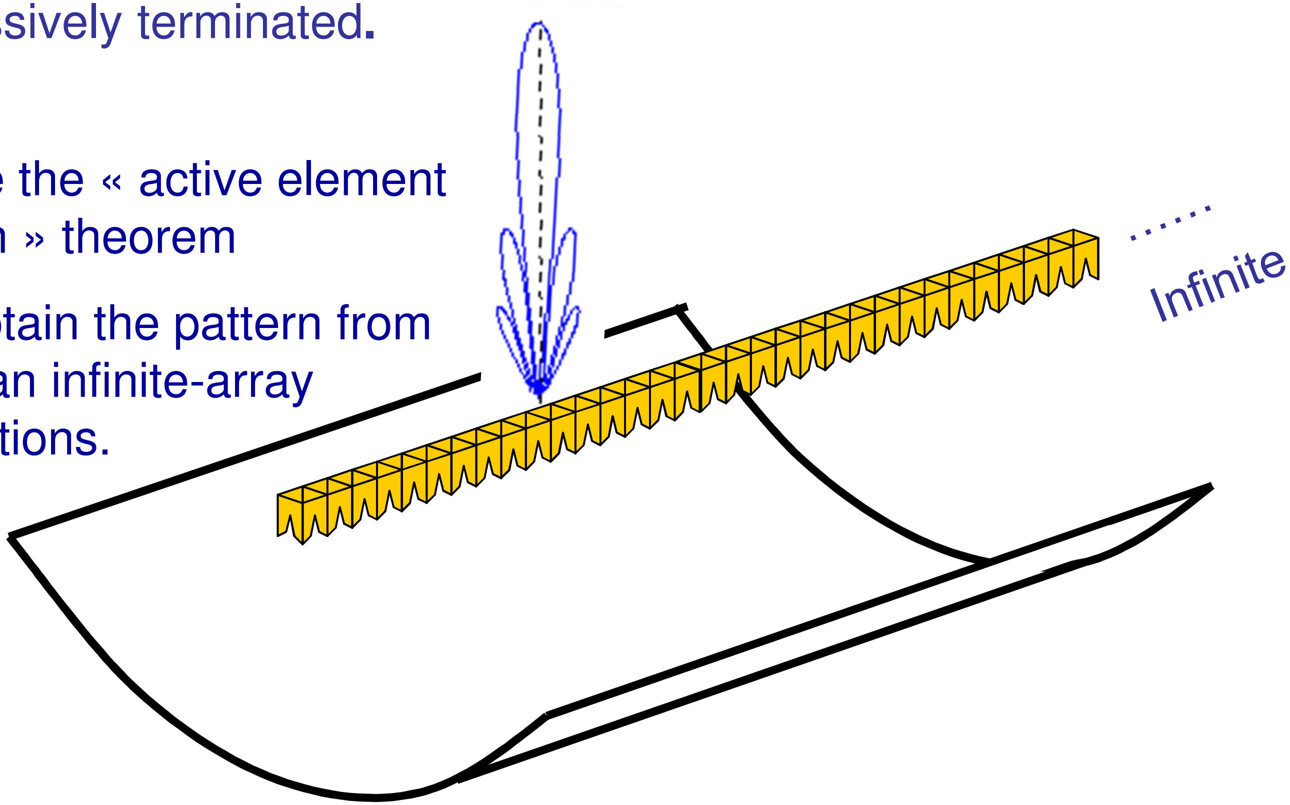
Element patterns

Compute the « **embedded element pattern** », i.e. the **secondary** pattern when one element is excited, the other ones are passively terminated**.**

⇒ Use the « active element pattern » theorem

i.e. obtain the pattern from full-scan infinite-array simulations.

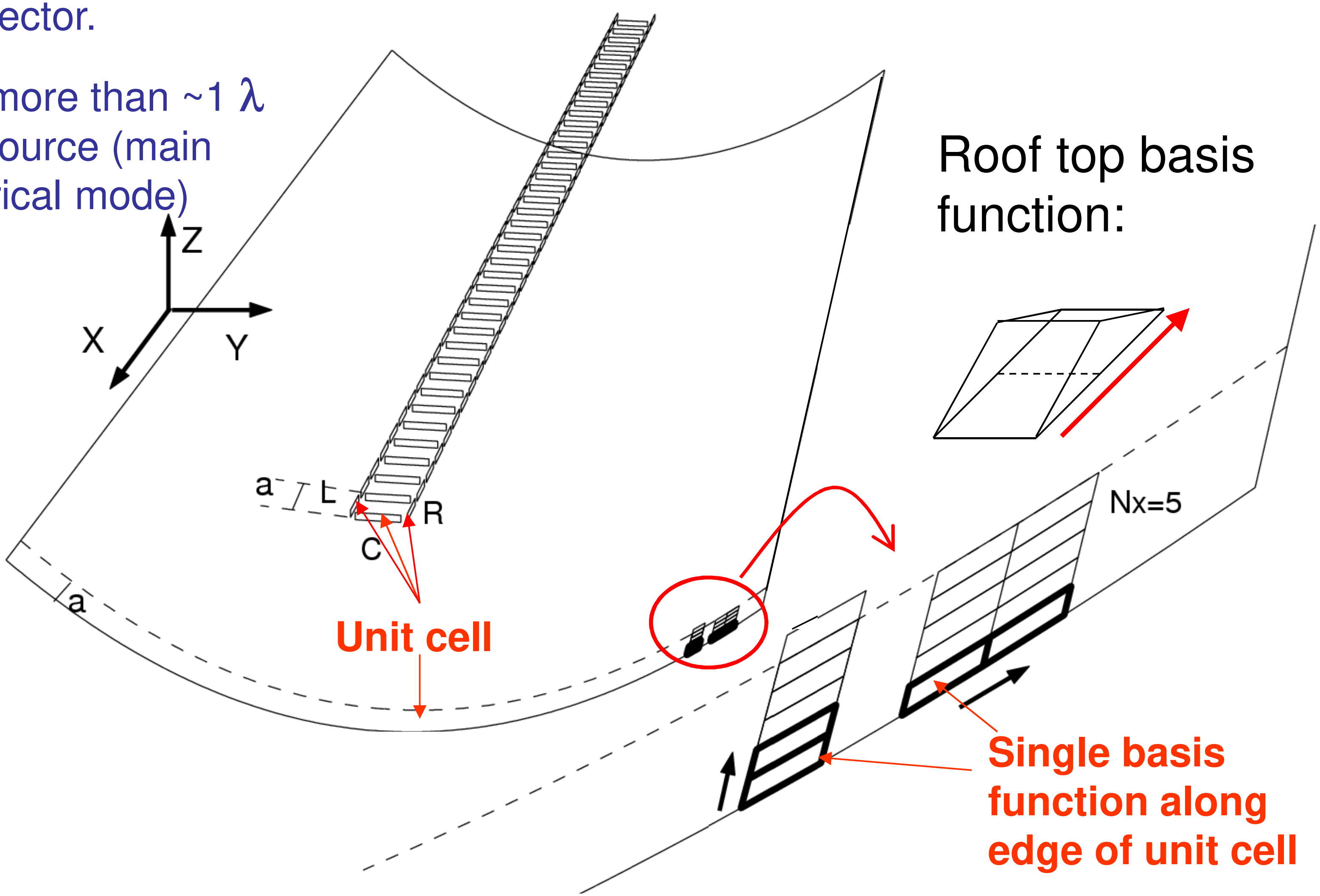


A « slice » of the cylinder is part of the unit cell

Reduction of unknowns

Hypothesis: linear phase progression of currents on reflector.

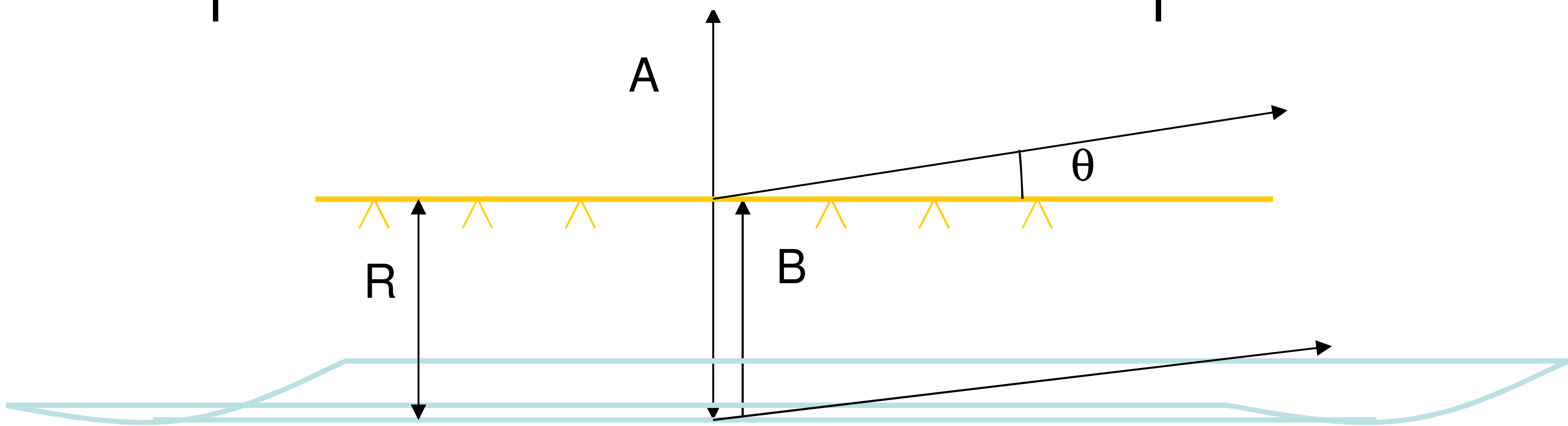
OK if more than ~1 λ from source (main cylindrical mode)



Array-reflector interaction

$$\left| A + B \overbrace{\exp\left(-2\, j\, k_0\, R\, \sin\theta\right)}^{\simeq H_0^{(2)}(2\,\gamma_0\, R)} \right|^2$$

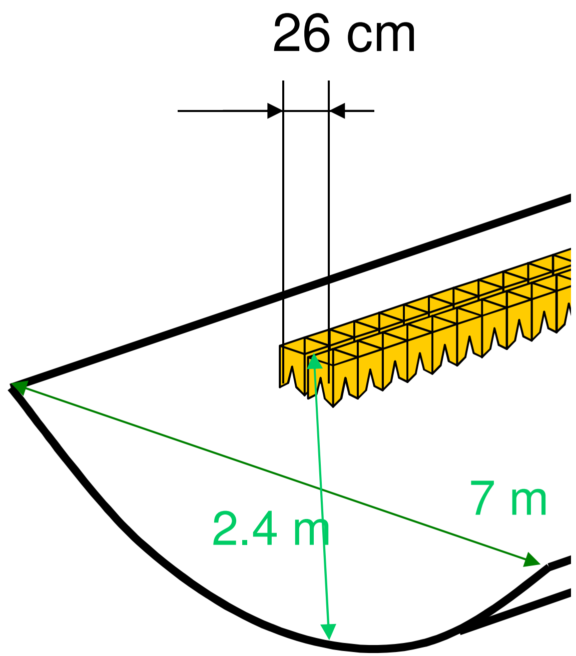
Hyp: A and B vary slowly with frequency

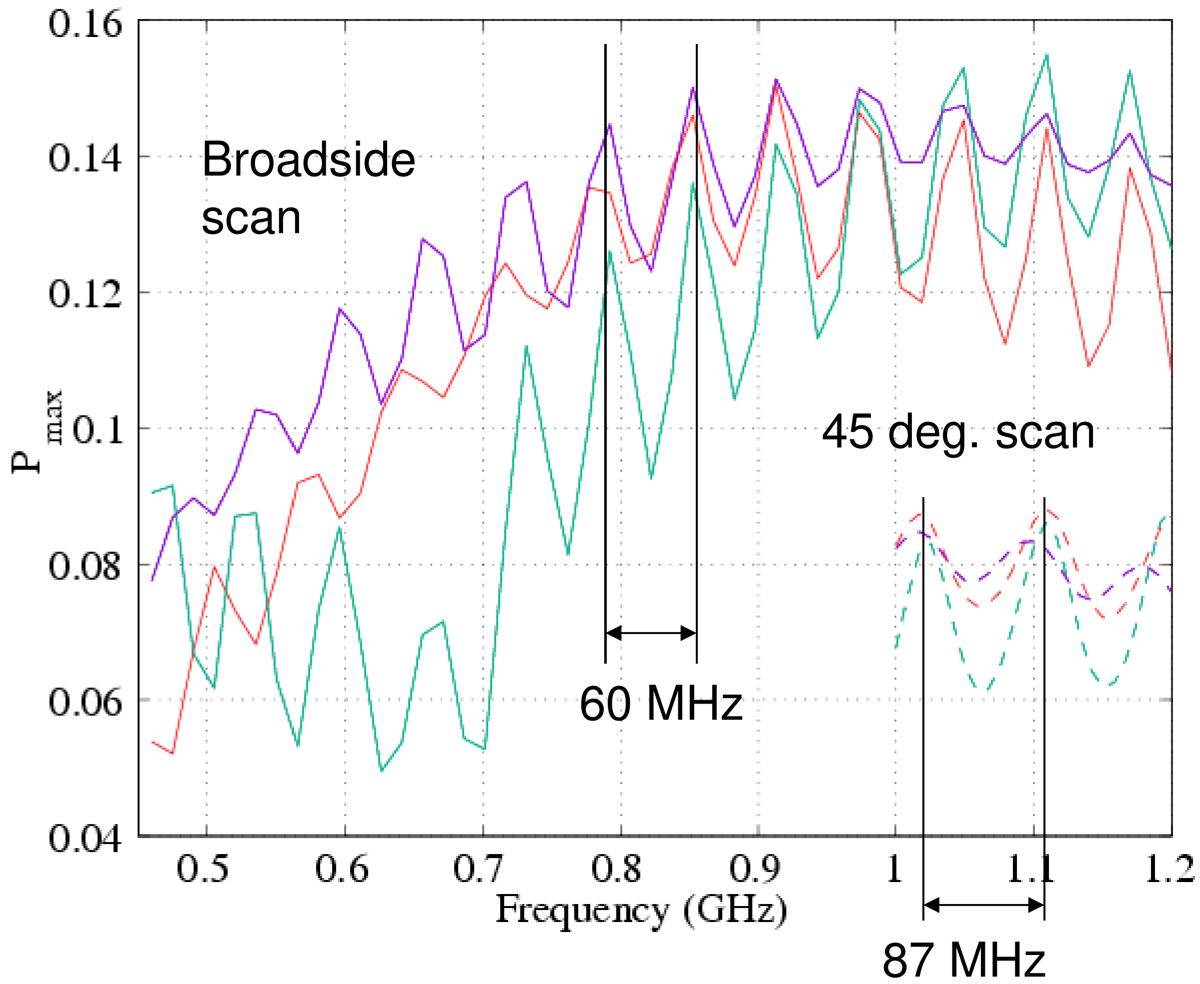


$$\cos(2\,k\,R\,\sin\theta) = \cos(2\,2\pi\, f/c\, R \sin\theta) \Longrightarrow \Delta_f = \frac{c}{2\,R \sin\theta}$$

See also: O.A. Iupikov, R. Maaskant, MV. Ivashina, A. Young, P-S Kildal, Fast and accurate analysis of reflector antennas with phased array feeds including multiple reflections between feed and reflector, IEEE T-AP, July 2014.

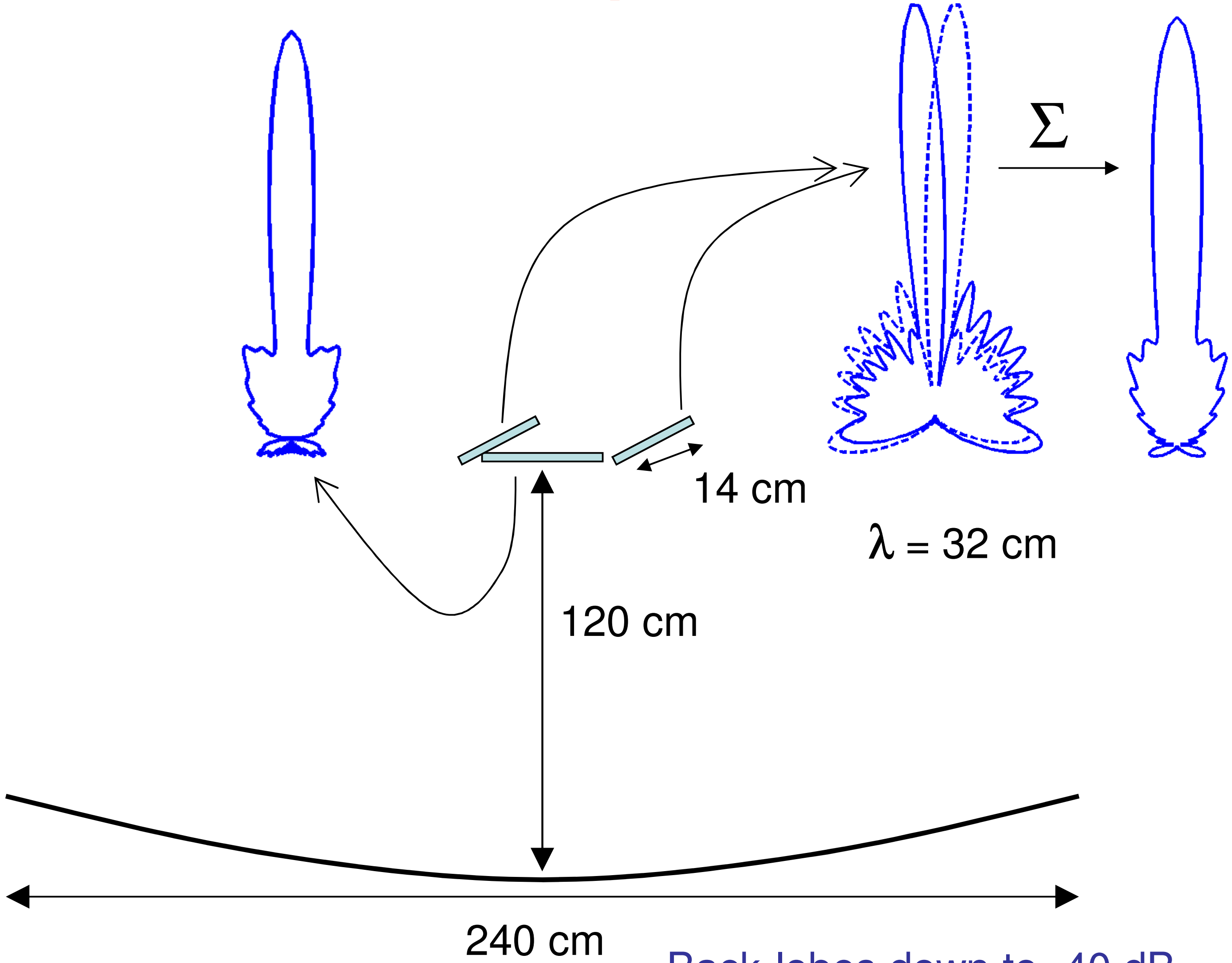
Oscillations in pattern maximum





~20 percent variations, while ratio of widths (of array and reflector) is near 4 percent only

Element patterns



Back-lobes down to -40 dB

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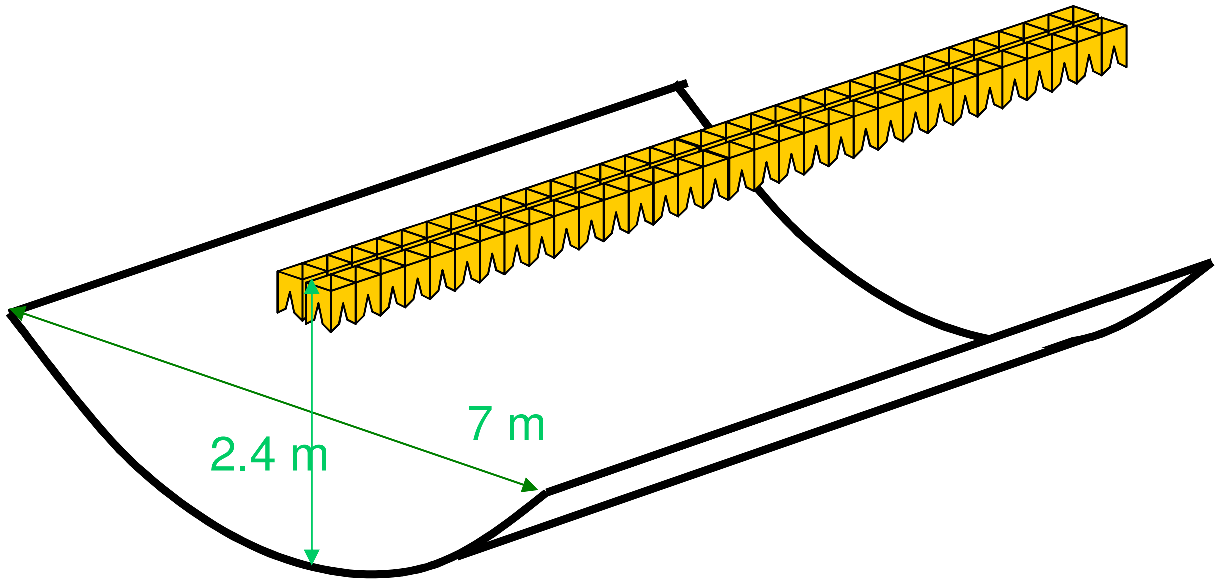
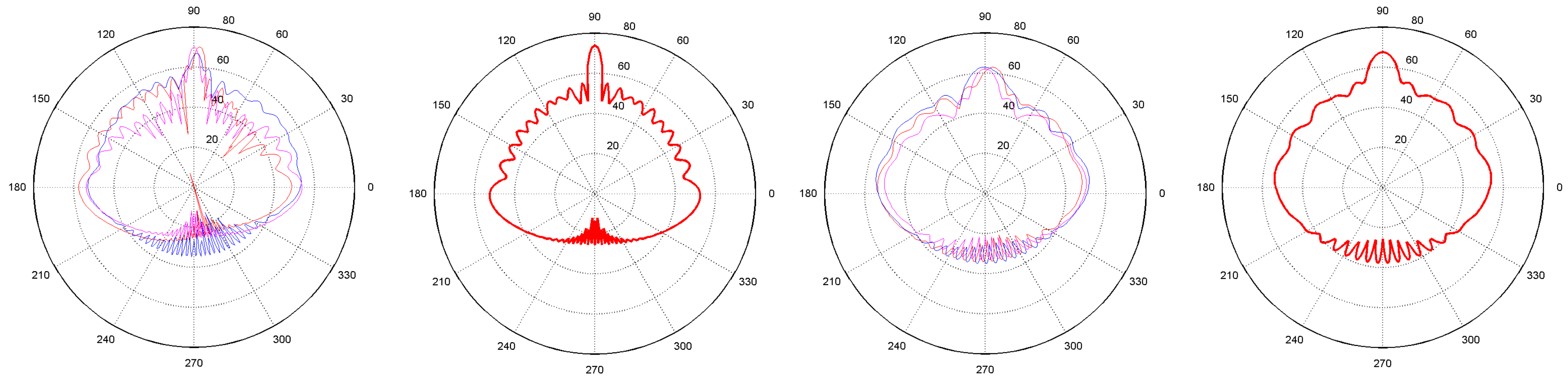
Combined patterns



Combined patterns

Five wideband tapered-slot antennas in each unit cell. Width of reflector: 710 cm. Focal length: 240 cm. Secondary patterns shown for antennas 1 (along axis), 2 (across axis) and 3 (along axis). They are shown for the array scanned at broadside and at 60 degrees from broadside. Red: patterns obtained when **combining the along-axis antennas** (1, 3 and 5), **with weights equal to 0.5, 1 and 0.5**.



Power balance

The power delivered to the active impedance of the antennas corresponds to the radiated power, plus the power lost in the loads terminating the other antennas located in the unit cell.

	radiated	dissipated	sum	delivered
Ant. 1	51.72	14.49	66.21	66.17
Ant. 2	71.58	42.19	113.77	113.76
Ant. 3	35.82	30.63	66.46	66.45

$$\mathcal{R}\{Z_a\}\ |I_a|^2/2 = \int_0^a \int_0^{2\pi} (|E_h|^2 + |E_v|^2)/(2\,\eta)\ d\phi\, dz \quad + \sum_{i=1}^{N} |I_i|^2\ Z_g/2$$

$$|E_{v,h}| = \sqrt{\frac{2}{\pi\, k_0\, R}\,\frac{k\,\eta}{4\,a}}\ \left|\int_{S_0} \vec{J}_S \,.\, \hat{n}_{v,h}\ dS_0\right|$$

Z_a is the active input impedance, I_a is the port current, E is the electric field, Z_g is the generator impedance, and a is the width of the unit cell.

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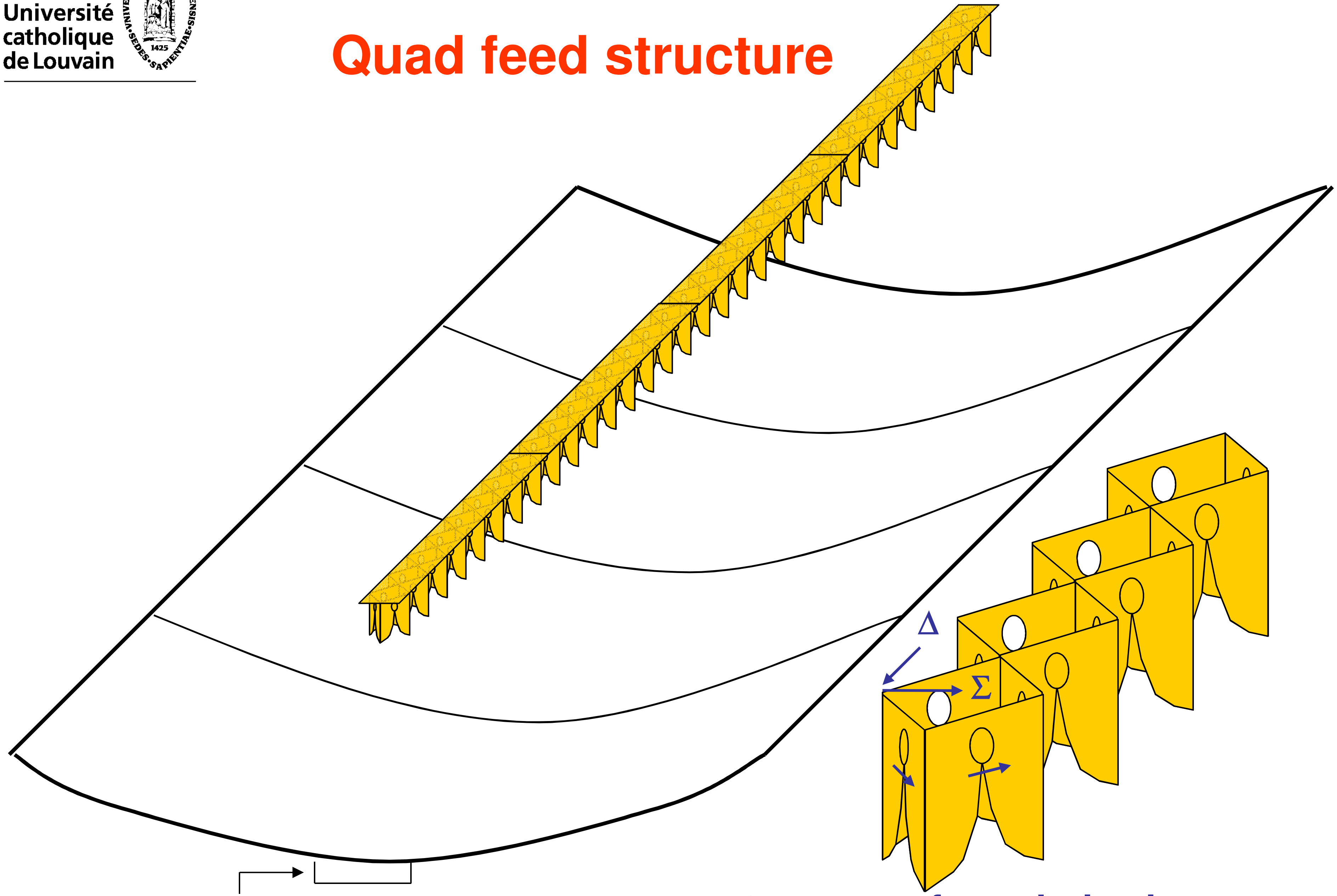
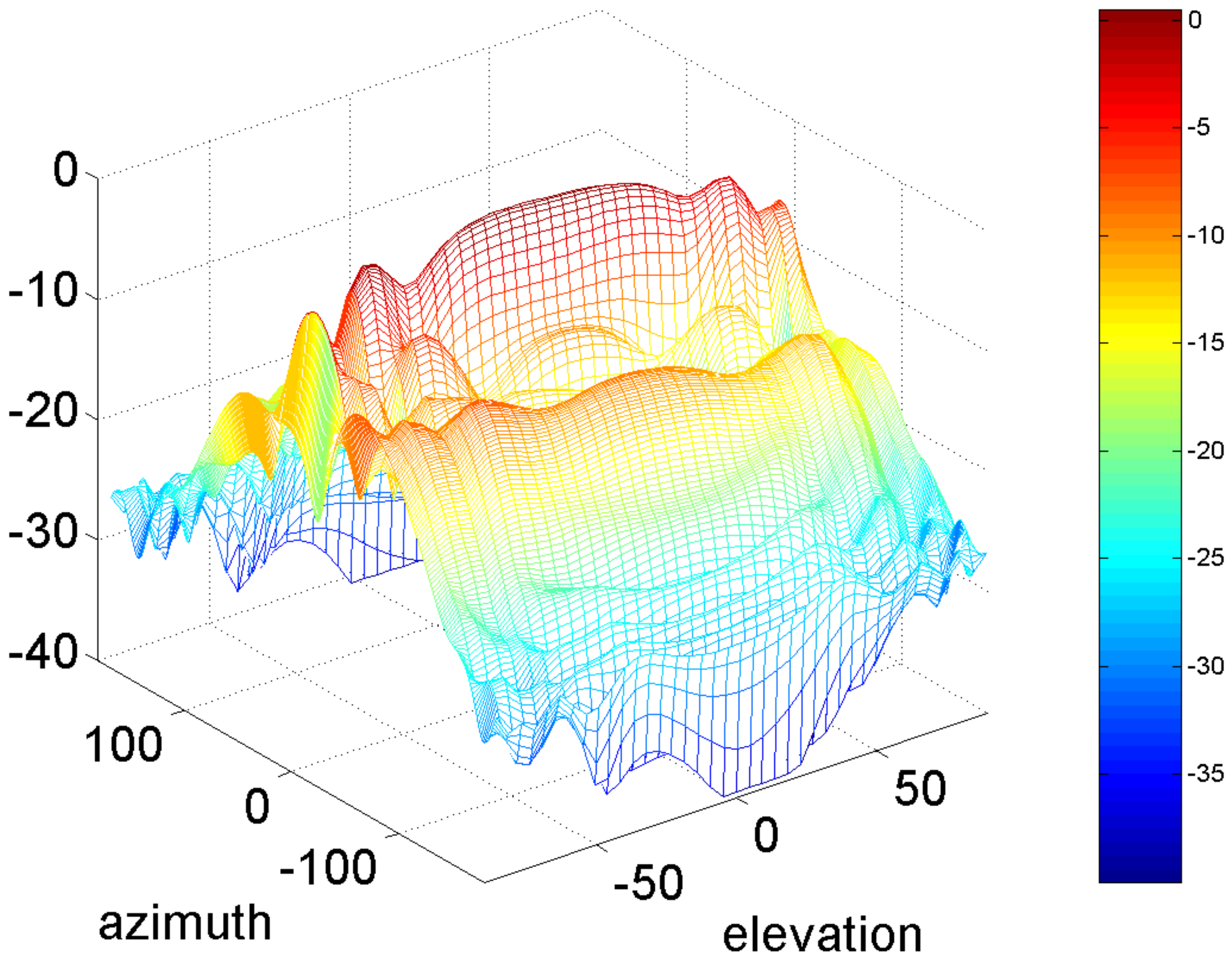
Quad feed structure



NB: vertex plate easily analyzed

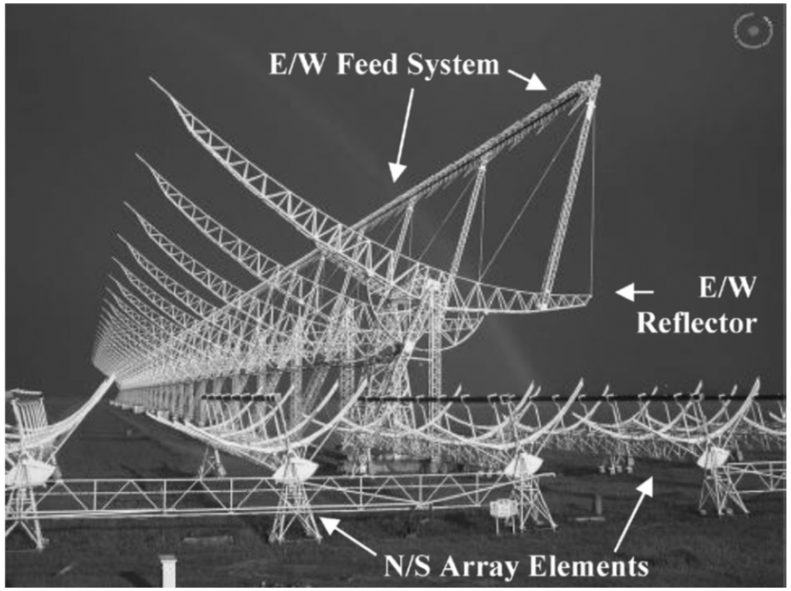
$\Sigma - \Delta$ strategy for polarization

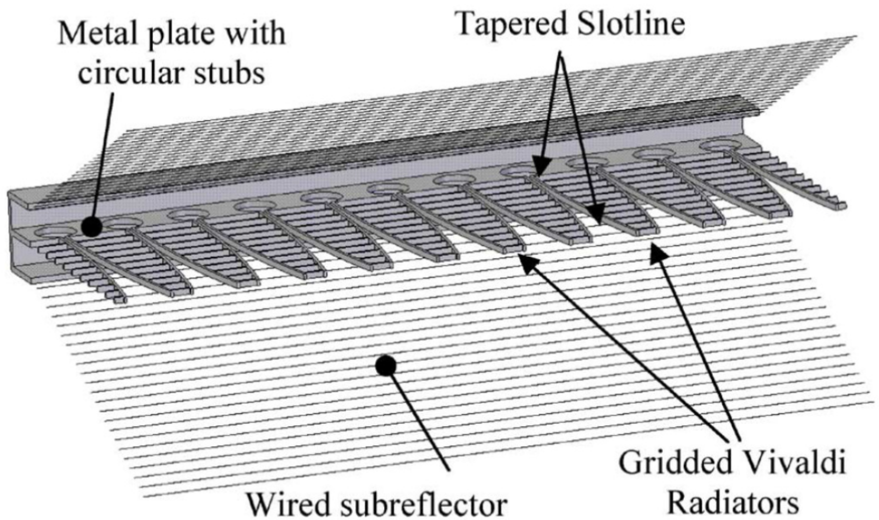
Element pattern of quad feed



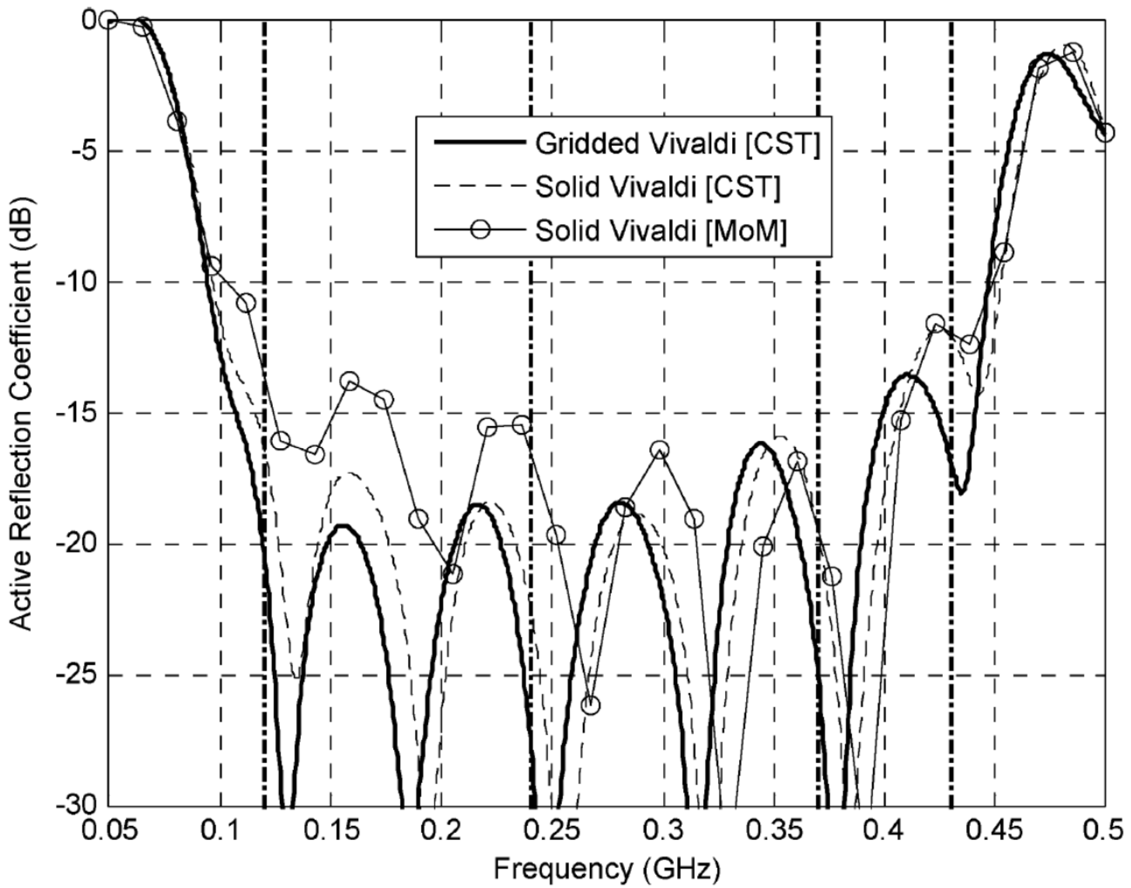
Northern-Cross telescope

G. Virone, R. Sarkis, C. Craeye, G. Addamo, O.A. Peverini, « Gridded Vivaldi antenna feed system for the Northern Cross radio telescope, » IEEE Trans. Antennas Propag., Vol. 59, pp. 2963-1971, June 2011.



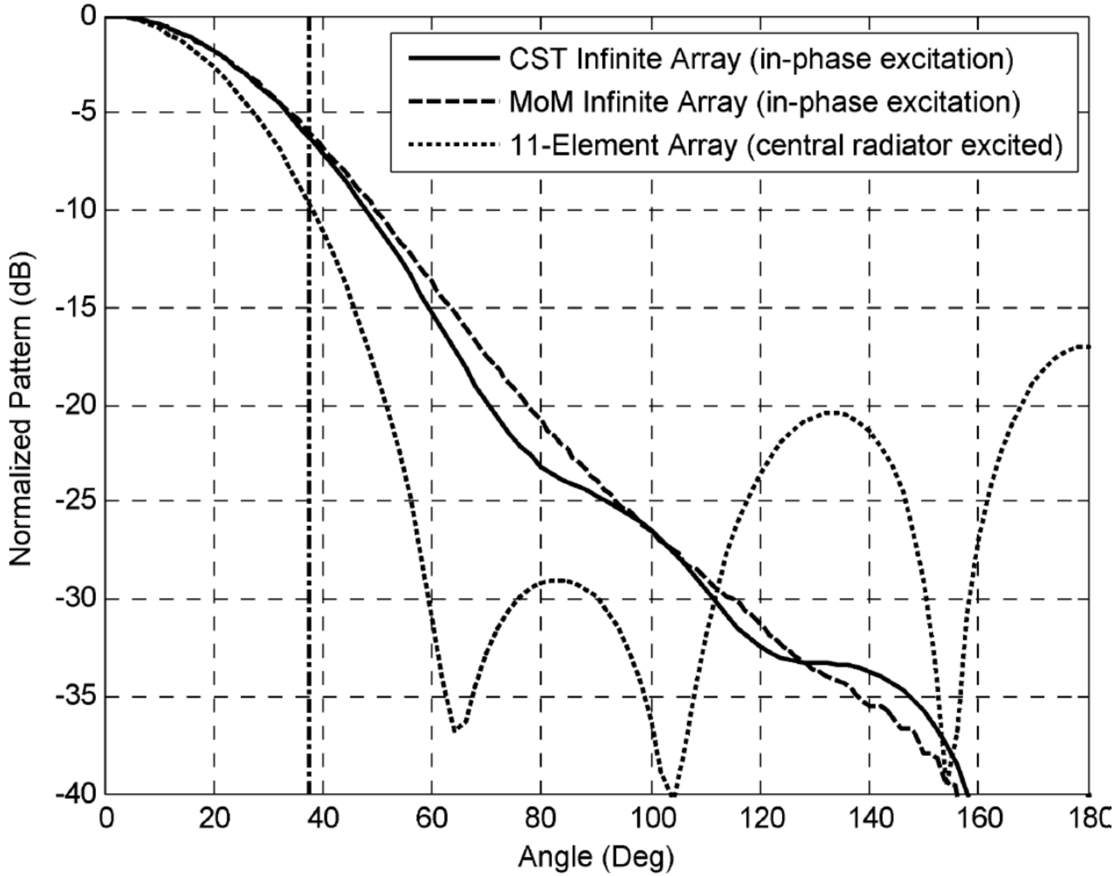


Active reflection coefficient



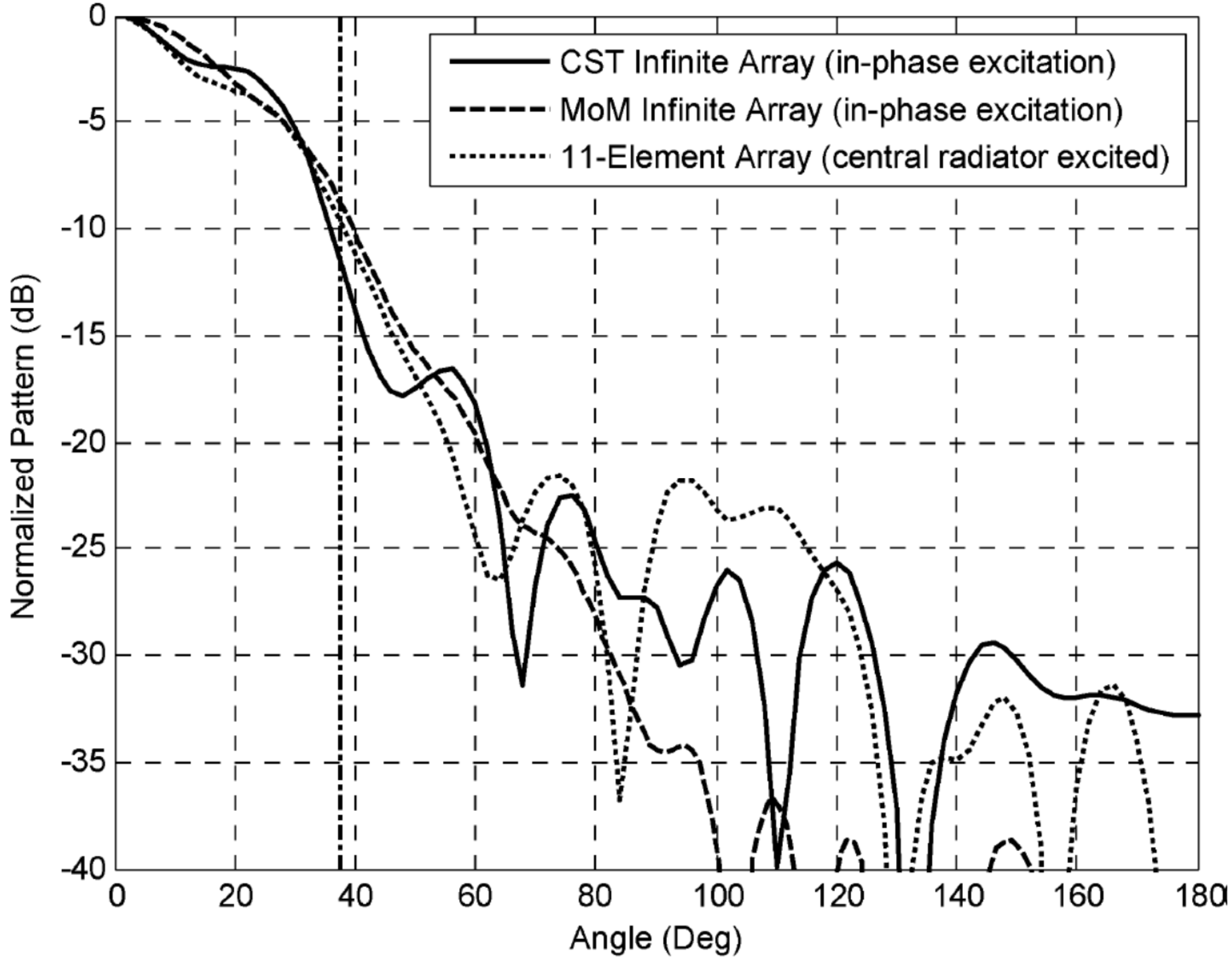
Magnitude of the active reflection coefficient for the designed antenna feed system (in-phase excitation). The vertical dash-dotted lines represent the LOFAR upper band 120–240 MHz and the Northern Cross 370–430 MHz enlarged band. The reference impedance is 90 Ohm.

H-plane pattern



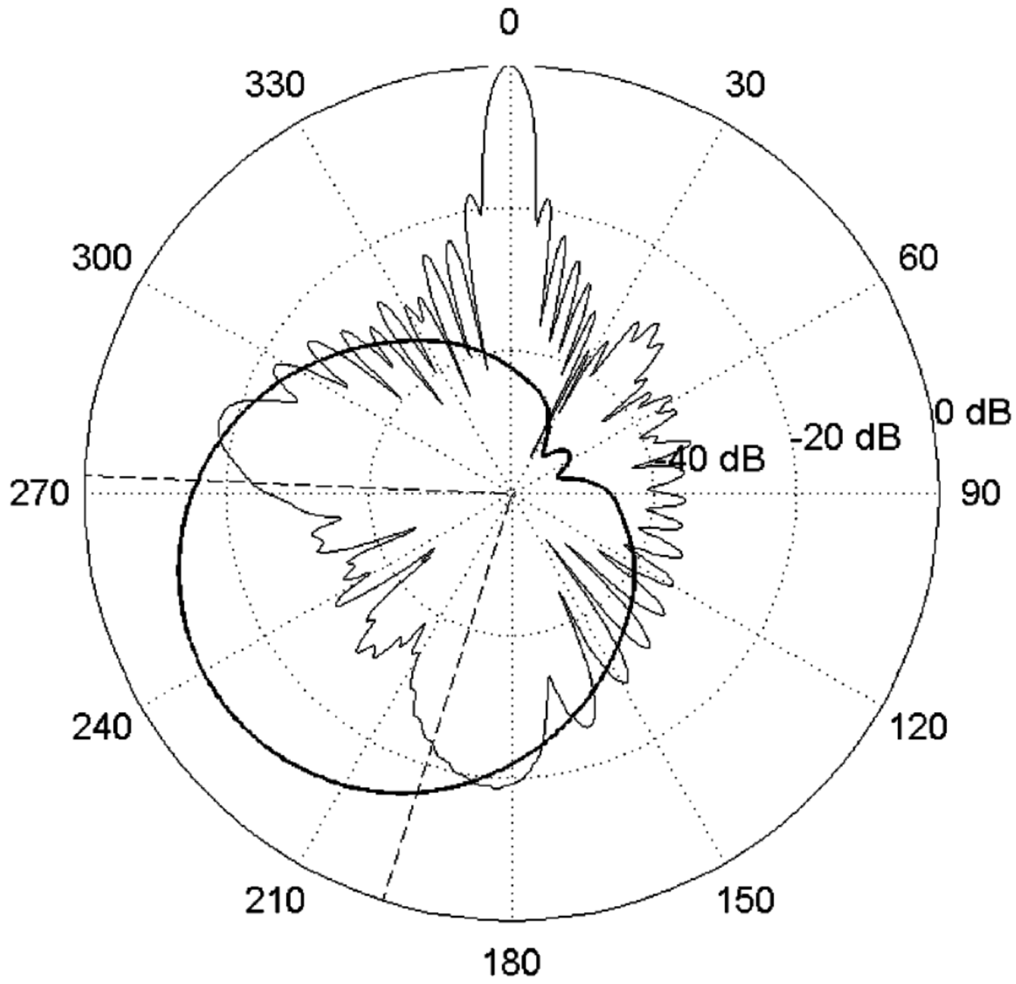
H-plane radiation pattern of the gridded Vivaldi array inside the subreflector at **120 MHz**. The black dash-dot line at 37.5 represents the main reflector edges.

H-plane pattern



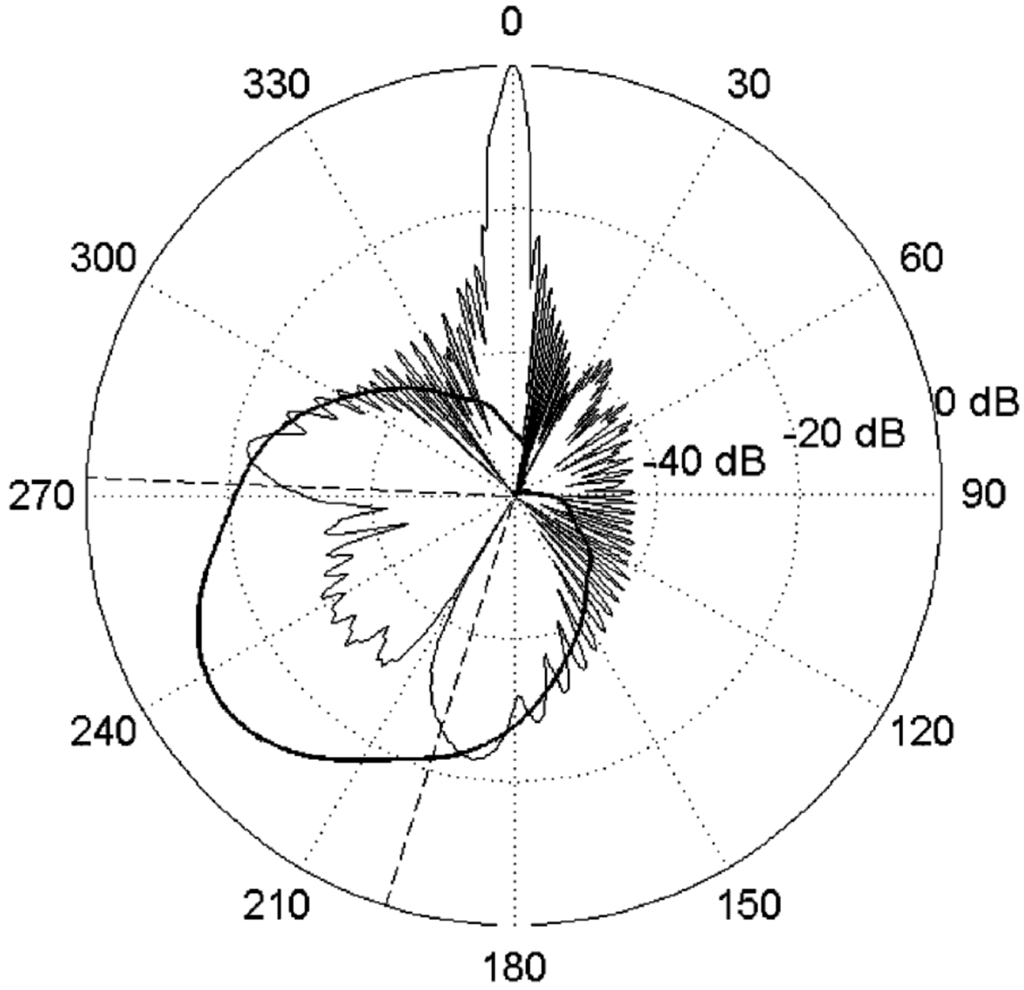
H-plane radiation pattern of the gridded Vivaldi array inside the subreflector at **408 MHz**. The black dash-dot line at 37.5 represents the main reflector edges.

Primary and secondary patterns



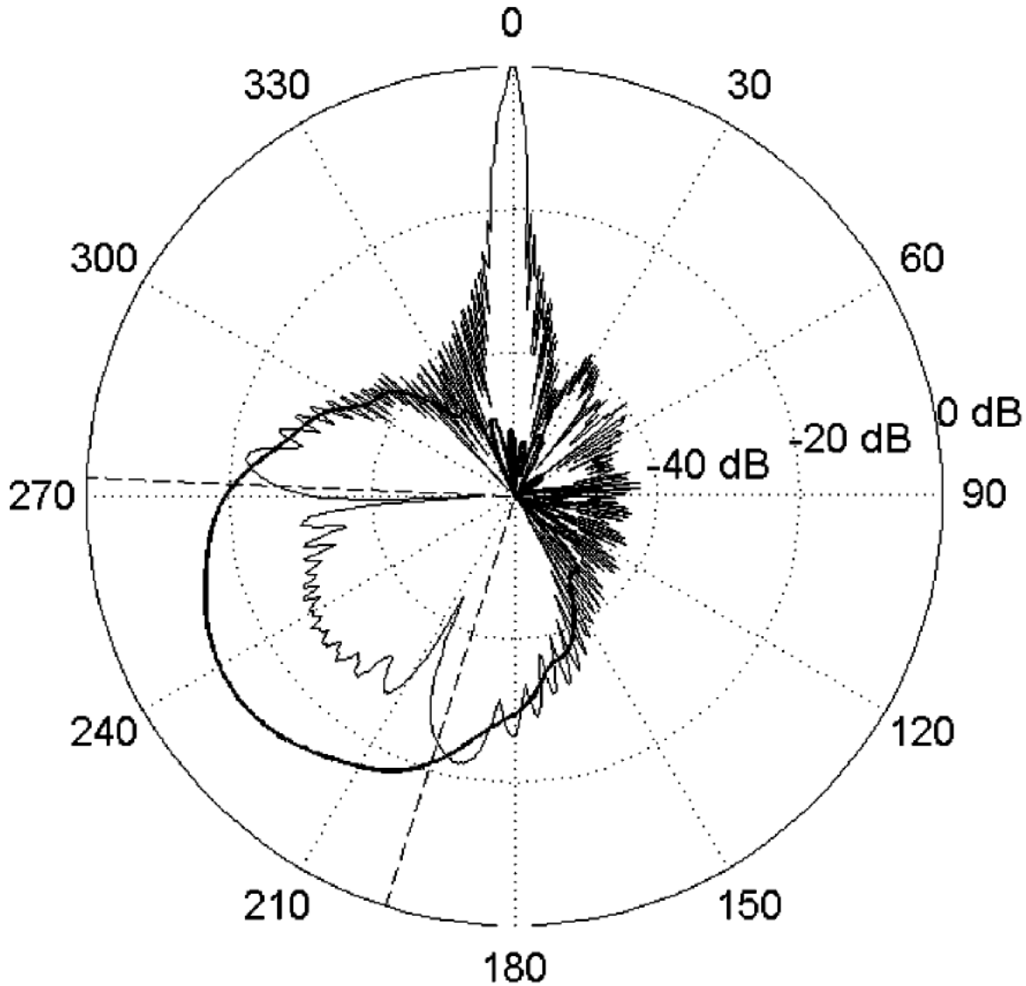
H-plane radiation pattern of the feed system (thick line) and the whole antenna system (thin line) at **120 MHz**. The dashed lines identify the main reflector edges.

Primary and secondary patterns



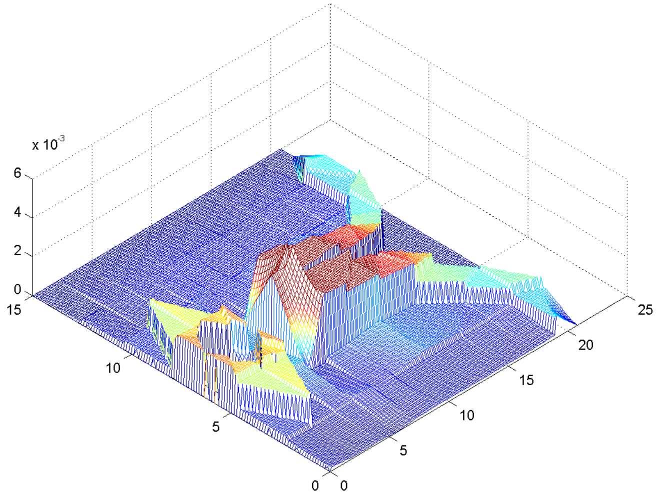
H-plane radiation pattern of the feed system (thick line) and the whole antenna system (thin line) at **240 MHz**. The dashed lines identify the main reflector edges.

Primary and secondary patterns

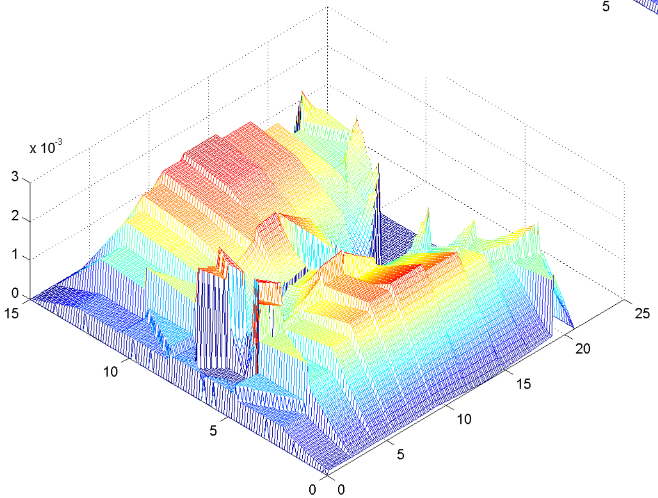


H-plane radiation pattern of the feed system (thick line) and the whole antenna system (thin line) at **408 MHz**. The dashed lines identify the main reflector edges.

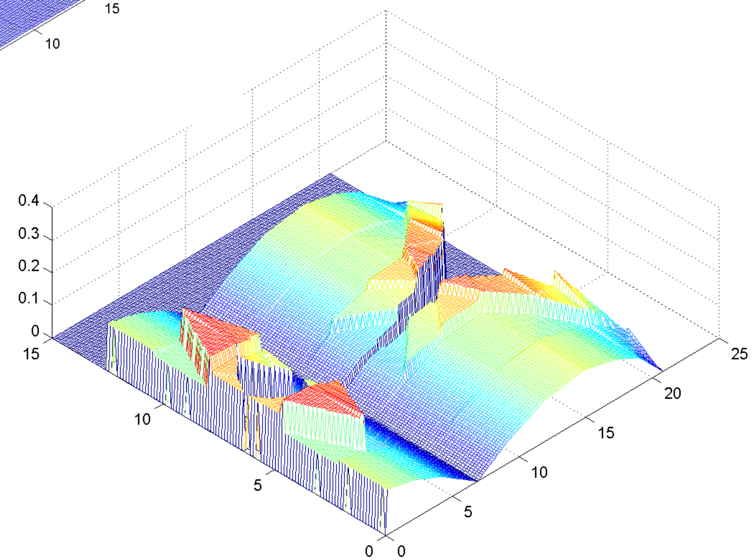
Analysis of anomalies



1 GHz

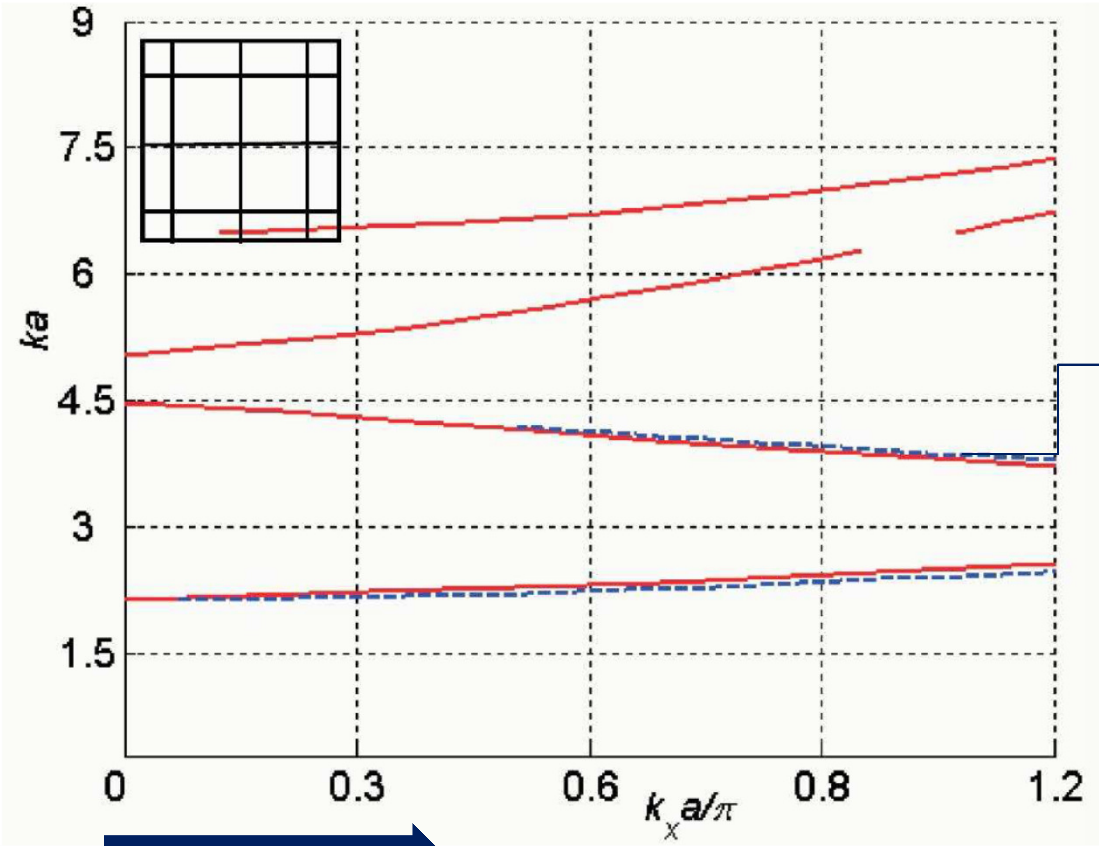


1.56 GHz anomalous currents at broadside



1.56 GHz, eigenvector with lowest eigenvalue

Eigenmodes and bandgaps



Frequency

Inter-element phase shift

Non-radiating eigenmode

Null in embedded pattern

Eigenmodes in a wire-medium, X. Dardenne, I. Nefedov, C. Craeye

Challenges

- Shift eigenmodes outside band of interest
- Analyze array/reflector truncation
- Introduce antenna “shaping” within unit cell